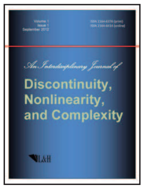

Discontinuity, Nonlinearity, and Complexity

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Title of Article

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Submission Info

Communicated by Referees
Received DAY MON YEAR
Accepted DAY MON YEAR
Available online DAY MON YEAR

Keywords

Keyword 1
Keyword 2
Keyword 3
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Abstract

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1 Section Heading

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Use the standard `equation` environment to typeset your equations, e.g.

$$\ddot{x}+\delta\dot{x}-\alpha x+\beta x^3=Q_0\cos\Omega t \tag{1}$$

where $\dot{x}=dx/dt$ is velocity. Q_0 and Ω are excitation amplitude and frequency, respectively. $\delta$is damping coefficient. α and β are linear and nonlinear stiffness coefficients of the Duffing oscillator.

however, for multiline equations we recommend to use the `eqnarray` environment[a].

$$\begin{aligned} a\times b &= c \\ \vec{a}\cdot\vec{b} &= \vec{c} \qquad (2) \end{aligned}$$

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[a]In physics texts please activate the class option `vecphys` to depict your vectors in ***boldface-italic*** type - as is customary for a wide range of physical subjects.

ISSN 2164 – 6376, eISSN 2164 – 6414/$-see front materials
DOI : 10.5890/DNC.2012.02.001

Theorem 1. *theorem*

Proposition 2. *theorem*

Lemma 3. *theorem*

Corollary 4. *theorem*

Conjecture 5. *theorem*

In mechanical engineering, in 1918, Duffing [1] presented the hardening spring model to describe the vibration of electro-magnetized vibrating beam. Since then, the Duffing oscillator has been extensively used to describe nonlinear structural vibrations in structural dynamics. In 1964, Hayashi [2] discussed the approximate periodic solutions and the corresponding stability by the averaging method and harmonic balance method. In 1973, Nayfeh [3] used the perturbation method to approximate periodic motion of the Duffing oscillators (also see, Nayfeh and Mook [4]). In 1979, Holmes [5] showed the strange attractors of chaotic motions in nonlinear oscillators via the Duffing oscillator with a twin-well potential. In 1980, Ueda [6] used numerical simulations to show chaotic motion via period-doubling of periodic motions of Duffing oscillators. In 1997, Luo and Han [7] analytically presented the stability and bifurcation conditions of periodic motions of the Duffing oscillator. The constant term of the analytical solution for the steady-state motion of the Duffing oscillator was not considered. In 1996, Luo and Han [8] presented an improved solution of the Duffing oscillator with a twin-well potential. For analytical prediction of chaos, in 1999, Luo and Han [9] investigated chaotic motions in nonlinear rod through the Duffing oscillator. For the periodically forced Duffing oscillator with damping, the analytical prediction of periodic solutions is still very difficult. In this paper, the analytical solutions of periodic motions will be investigated and the analytical route of periodic motions to chaos will be of great interest.

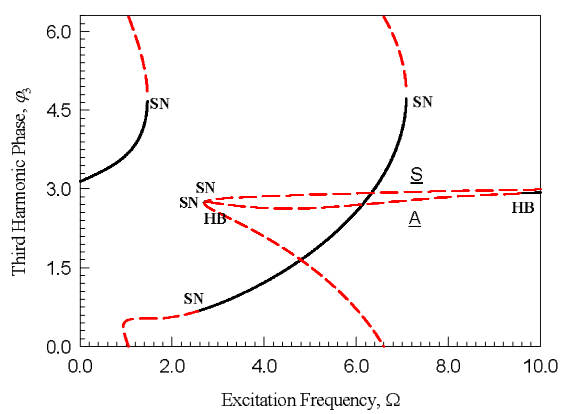

Fig. 1 The analytical prediction of periodic solutions based on two harmonic terms (HB3): (a) constant term a_0; (b)-(d) harmonic amplitudes $A_k(k=1,2,3)$; and (e)-(f) harmonic phases φ_k $(k=1,2)$ for right potential well. $(\delta=0.5, \alpha=-10.0, \beta=10.0, Q_0=10.0)$.

To look for approximate analytical solution of nonlinear oscillator, such an issue started from Lagrange [10] to investigate the three-body problem as a perturbation of the two-body problem by the method of averaging. In the end of the 19th century, Poincare [11] further developed the perturbation theory to investigate the motions of celestial bodies. In 1920, van der Pol [12] used the method of averaging to determine the periodic solutions of oscillation systems in circuits. Until 1928, the asymptotic validity of the method of averaging was not proved. In 1928, Fatou [13] gave the proof of the asymptotic validity through the solution existence theorems of differential equations. In 1935, Krylov, Bogoliubov and Mitropolsky [14] further developed the method of averaging, and the detailed presentation was given. In 1964, Hayashi [2] presented the perturbation methods including averaging method and principle of harmonic balance. In 1969, Barkham and Soudack [15] extended the Krylov-Bogoliubov method for the approximate solutions of nonlinear autonomous second order differential equations (also see, Barkham and Soudack [16]). In 1987, a generalized harmonic balance approach was used by Garcia-Margallo and Bejarano [17] to determine approximate solutions of nonlinear oscillations with strong nonlinearity. In the same year, Rand and Armbruster [18] used the perturbation method and bifurcation theory to determine the stability of periodic solutions. In 1989, Yuste and Bejarano [19] used the elliptic functions rather than trigonometric functions to improve the Krylov-Bogoliubov method. In 1990, Coppola and Rand [20] used the averaging method with elliptic functions to determine approximation of limit cycle. In 1997, Luo and Han [7] analytically studied the stability and bifurcations of periodic solutions of Duffing oscillators through the first order harmonic balance method, and provided the analytical conditions for the Hopf and saddle-node bifurcations. To obtain accurate results of periodic solutions in nonlinear vibration, many harmonic terms are included in the harmonic balance method. In 2008, Peng et al [21] presented the approximate period-1 solution for the Duffing oscillator by the HB3 method compared with the fourth-order Runge-Kutta method. In 2011, Luo and Huang [22] further discussed a generalized harmonic balance method to obtain the analytical solution of period-1 motion. Luo and Huang [23] also presented a generalized harmonic balance method to determine period-*m* solutions in nonlinear oscillators.

In this paper, the generalized harmonic balance method will be used to investigate analytical periodic motions in the periodically forced Duffing oscillator with a twin-well potential. The bifurcation tree from period-1 motions to chaos will be presented with varying parameters. The corresponding unstable periodic motions in the Duffing oscillator will be presented for a better understanding of nonlinear dynamics in such a Duffing oscillator. Numerical illustrations of stable and unstable periodic motions will be carried out.

2 Section Heading

From Eq.(1), the standard form is

$$\ddot{x} + f(x, \dot{x}, t) = 0 \tag{3}$$

2.1 Subsection Heading

The Fourier series expression of any periodic motion in nonlinear systems needs infinite terms to give the exact solution of such a periodic motion. In practice, it is impossible to do so. Thus, the truncated Fourier series solutions will be used to give an approximate solution that can be close to the exact solution. From such approximate, analytical solutions, the equilibrium solution of coefficient dynamical system for the Fourier series of the periodic motion can be obtained from Eq.(3) using Newton-Raphson method, and the stability and bifurcation analysis of the such equilibrium points can be completed through the eigenvalue analysis. The system parameters are

$$\delta = 0.5, \alpha = -10.0, \beta = 10, Q_0 = 10.0 \tag{4}$$

The backbone curves of harmonic amplitude varying with excitation frequency Ω are illustrated. The harmonic amplitude and phase are defined by

$$A_{k/m} \equiv \sqrt{b_{k/m}^2 + c_{k/m}^2}, \varphi_{km} = \arctan \frac{c_{k/m}}{b_{k/m}} \tag{5}$$

and the corresponding solution in Eq.(43) is

$$x^*(t) = a_0^{(m)} + \sum_{k=1}^{N} A_{k/m} \cos\left(\frac{k}{m}\Omega t - \varphi_{k/m}\right). \tag{6}$$

In Luo and Han [8], one term harmonic term was considered for period-1 motions for the large and small orbit. In this paper, many harmonic terms will be considered to achieve a more accurate prediction of the periodic motions. For period-1 motion, the first three harmonic terms of the Fourier series expansion (HB3) will be used to obtain the approximate periodic solutions. The constant term $a_0^{(1)} \equiv a_0$ and the first three harmonic amplitudes A_k and phases $\varphi_k (k = 1, 2, 3)$ versus excitation frequency are plotted in Fig.1(a)-(g), respectively. A parameter map is presented in Fig.2.

The initial conditions for stable period-1 motion $(\Omega = 2.75)$, unstable period-1 motion and stable period-2 motion $(\Omega = 2.753)$, unstable period-1 motion, unstable period-2 motion and stable period-4 motion $(\Omega = 2.7537)$ are listed in Table 1.

Table 1 Input data for numerical simulations of periodic motions $(\delta = 0.5, \alpha = -10.0, \beta = 10.0, Q_0 = 10.0)$

	Ω	Initial conditions $(t = 0.0)$		Stability	Period-m
		x_0	$\dot{x}_0$		
Fig.10(a)	2.75	−0.724512	0.251206	Stable(HB)	Period-1(HB5)
Fig.10(b)	2.75	−0.724512	0.251206	Stable(HB)	Period-1(HB5)
Fig.10(c)	2.75	−0.724512	0.251206	Stable(HB)	Period-1(HB5)
Fig.10(d)	2.75	−0.724512	0.251206	Stable(HB)	Period-1(HB5)
Fig.10(e)	2.75	−0.724512	0.251206	Stable(HB)	Period-1(HB5)
Fig.10(f)	2.75	−0.724512	0.251206	Stable(HB)	Period-1(HB5)

3 Conclusions

In this paper, analytical routines of period-1 motions to chaos in the Duffing oscillator with a twin-potential well were discussed comprehensively through the generalized harmonic balance method. The analytical solutions of period-*m* motions were developed by the Fourier series and the corresponding Hopf bifurcations of periodic motions cause new periodic motions with period-doubling. Three analytical routes of asymmetric period-1 motions to chaos were developed. The approximate, analytical periodic solutions were verified via numerical simulations, and the analytical, unstable periodic motions were given as well. With exact unstable periodic motion, the numerical simulations should stay with the analytical solution if without any computational errors. The analytical routes with unstable periodic motions can lead us to find unstable chaos.

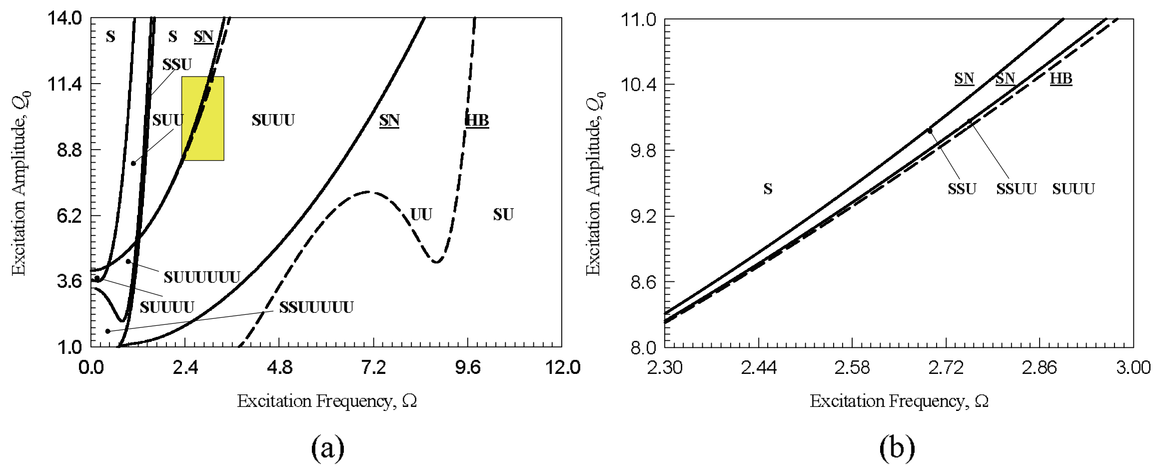

Fig. 2 A parameter map from the analytical prediction of periodic solutions based on three harmonic terms (HB3): (a) Global view and (b) zoomed view. ($\delta = 0.5, \alpha = -10.0, \beta = 10, Q_0 = 10.0$).

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