EXPERIMENTAL PARAMETERS ESTIMATION OF SATELLITE ATTITUDE CONTROL SIMULATOR

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Abstract: *Placing a satellite or any other spacecraft in orbit is a risky and expensive process; years of research and a lot of money are transformed into equipments that will be beyond any possibility of maintenance in case something goes wrong. Therefore, space projects must be carried on as carefully as possible in order to guarantee that satellite equipments perform its mission properly. In that context, experimental validation of new equipment and/or control techniques through prototypes is the way to increase system confidence. The Space Mechanics and Control Division (DMC) of INPE is constructing a 1D simulator, with rotation around the vertical axis and a 3D simulator, with rotation in three axes, to implement and test satellite Attitude Control System (ACS). However, to perform experimental test it is necessary to estimate the platform inertia parameters in order to balance the platform accurately, so it behaves similar to space torque free conditions. This paper presents the equations of motion and control law design for a 3 D attitude control system simulator. This 3 D model is simplified to 1D simulator from which the inertia moment is estimated by a recursive least squares that uses experimental data. The platform data are obtained in a simple experiment where a reaction wheel is used to apply torques and a gyroscope is used to measure the platform angular velocity. The inertia moment estimated by this approach is very close to the platform inertia moment value obtained by other method.*

Keywords: *experimental procedure, inertia estimation, satellite attitude control*

1 Introduction

There are several methodologies to investigate the satellite ACS performance, depending on the investigation objectives. Computer simulations cannot be the appropriate one. The use of experimental platforms have the important advantage of allowing the satellite dynamics representation in laboratory and once validated this platform, it is possible to accomplish experiments associated with simulations to evaluate satellites ACS with simple rigid dynamics as well as complex configurations involving flexible components. The preference for using experimental test is associated to the possibility of introducing more realism than the simulation, however, it has the difficulty of reproducing zero gravity and torque free space condition, extremely relevant for satellites with complex dynamics and ACS with great degree of precision. Experimental platforms with rotation around three axes are more complicated assemblies than rotation around one axis, but it is more representative. Examples of the use of experimental platforms for investigating different aspects of the satellite dynamic and control system can be found in (Hall, et al., 2002 and Berry, et al., 2003). A classic case of a phenomenon with no experimental investigations before launch was the dissipation energy effect that has altered the satellite Explorer I rotation (Kaplan, 1976). A pioneering experiment to study energy dissipation effect has been done by Peterson (1976). An important aspect that is possible to investigate through experimental platforms is the satellite inertia parameters identification. An initial way to estimate the inertia parameters is using the CAD software. The results obtained by CAD can be compared to the results obtained through estimation techniques. The least square method with batch processing was used with success by the satellite simulator developed by Ahmed, et al. (1998) and Tanygin and Williams (1997). For platforms where the dynamic has variation in time, like, mass center and inertia parameters variation, the application of parameters estimation methods in real time becomes more appropriate. Lee and Wertz, (2002) has developed an algorithm based on the least square method to identify mass parameters of a space vehicle in rotation during attitude maneuvers. Methods with the same objectives, but based on Kalman filter theory were used by Bergmann and Dzielski (1990). Experimental platforms can also be used for familiarizing with modeling aspect and controllers designed for complex space structures (Dichmann and Sedlak, 1998). An experimental apparatus was used in (Cannon and Rosenthal, 1984) to investigate the dynamics and the control laws for a satellite composed of rigid and flexible parts. The results showed that the control of the flexible structure is extremely sensitive to the variation of the parameters of the system, indicating the need of more robust control strategies (Souza, 1992) in order to improve the controller performance. A detailed model with the equations of motions and control law design for a satellite simulator has been done by Souza (2007).

2 Equations of motion for the simulator

The 3D simulator of INPE with rotation in three axes and the 1D simulator with rotation in the vertical axis are shown in Fig. 1. Both simulators have a disk-shaped platform, supported on a plane and a spherical air bearing, respectively. The platform can accommodate various satellites components; like sensors, actuators, computers and its respective interface and electronic. However, at the moment only the 1D is equipped with a sensor, an actuator, a computer, a battery and its respective interfaces.

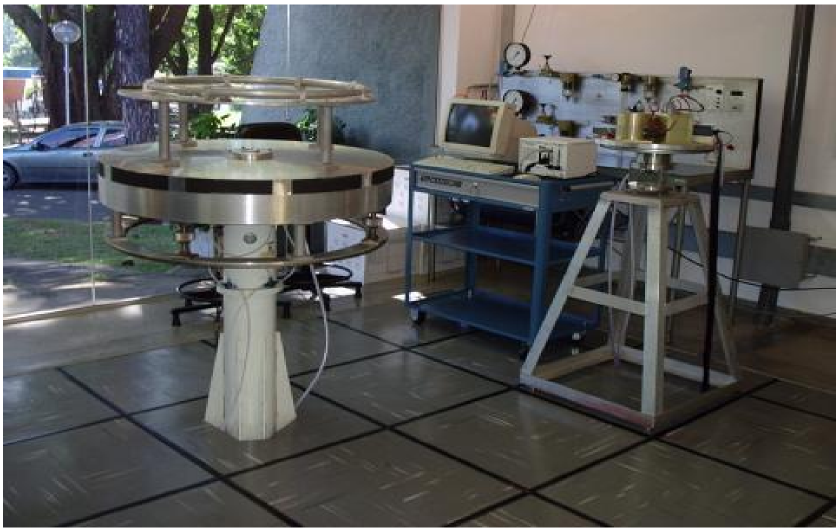

Figure 1. INPE 3D and 1D Satellite Attitude Control System Simulators.

The 3D simulator equations of motion are derived considering the coordinates system (x, y, z) that is fixed to the platform with origin in its rotation center and the coordinate systems $(x, y, z)_{1,2,3}$ fixed to the reaction wheels 1, 2 and 3 with origin in their respective mass centers and aligned with their rotation axes, as shown Fig. 2. The vectors $R_{1,2,3}$ indicate the reaction wheels position and the vector $\vec{r}$ locates the platform elements of mass dm with respect to (xyz). The vectors $\rho_{1,2,3}$ locates the reaction wheels elements of mass $dm_{1,2,3}$ with respect to the coordinates $(x,y,z)_{1,2,3}$. The reaction wheels angular velocity are $\vec{w}_1$, $\vec{w}_2$, $\vec{w}_3$; and the platform angular velocity is given by:

$$\vec{W} = p\vec{i} + q\vec{j} + r\vec{k} \tag{1}$$

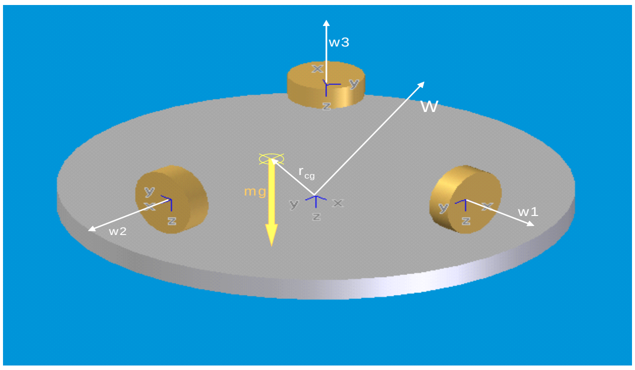

Figure 2. Satellite Attitude Control System Simulator Platform.

The platform angular moment is the sum of the base and reaction wheels angular moment given by:

$$\vec{H} = \int_B (\vec{r} \times \vec{v})dm + \sum_{i=1}^{3} \int_{RW} (\vec{r} \times \vec{v})dm \tag{2}$$

where the angular velocity of the base is $\vec{v} = \vec{W} \times \vec{r}$ and the reaction wheels is $\vec{v} = \vec{W} \times \vec{R}_i + \vec{w}_i \times \vec{\rho}_i$. Introducing the last two velocities expression into Eq.(2) one has

$$\vec{H} = \int_{B+RW} \vec{r} \times (\vec{W} \times \vec{r}) dm + \sum_{i=1}^{3} R \int_{RW} \vec{\rho}_i \times (\vec{w}_i \times \vec{\rho}_i) dm \tag{3}$$

which in compact form can be represented by

$$\vec{H} = \vec{h} + \sum_{i=1}^{3} \vec{h}_i \tag{4}$$

The equations of motion of the platform is obtained by deriving the total angular moment and equalizing to the external torques acting on the platform which is given by

$$\vec{r}_{cg} \times (m\vec{g}) = \frac{d\vec{H}}{dt} = (\dot{\vec{h}})_r + \vec{W} \times \vec{h} + \sum_{i=1}^{3} (\dot{\vec{h}}_i)_r + \vec{W} \times \left(\sum_{i=1}^{3} \vec{h}_i \right) \tag{5}$$

$\vec{r}_{cg}$ represents the center of gravity location where the gravitational force (mg) acts.

Applying the same principle, the reaction wheels equations of motion are given by

$$\begin{aligned} T_1 &= I_1 [\dot{w}_1 + \dot{p}] \\ T_2 &= I_2 [\dot{w}_2 + \dot{q}] \\ T_3 &= I_3 [\dot{w}_3 + \dot{r}] \end{aligned} \tag{6}$$

where $T_{1,2,3}$ and $I_{1,2,3}$ are the control input and inertia moments of the reaction wheels.

The kinematics equations in terms of Euler angles (ϕ, θ, ψ) in the sequence 3-2-1 are given by

$$\begin{aligned} \dot{\phi} &= p + \tan(\theta)[q sin(\phi) + r\cos(\phi)] \\ \dot{\theta} &= q\cos(\phi) - r sin(\phi) \\ \dot{\psi} &= \sec(\theta)[q sin(\phi) + r\cos(\phi)] \end{aligned} \tag{7}$$

Putting Eq. (6) and Eq.(7) together with respect to the same coordination system yields

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} & 0 & 0 & 0 & I_1 & 0 & 0 \\ I_{xy} & I_{yy} & I_{yz} & 0 & 0 & 0 & 0 & I_2 & 0 \\ I_{xz} & I_{yz} & I_{zz} & 0 & 0 & 0 & 0 & 0 & I_3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{Bmatrix} = \begin{Bmatrix} (I_{xx} - I_{zz})(qr) + I_{xy}(pr) - I_{xz}(pq) + I_{yz}(r^2 - q^2) + I_2(w_2 r) + \\ - I_3(w_3 q) + mgr_y \cos(\phi)\cos(\theta) - mgr_z sin(\phi)\cos(\theta) \\ (I_{zz} - I_{xx})(pr) + I_{yz}(pq) - I_{xy}(qr) + I_{xz}(p^2 - r^2) - I_1(w_1 r) + \\ + I_3(w_3 p) - mgr_x \cos(\phi)\cos(\theta) - mgr_z sin(\theta) \\ (I_{xx} - I_{yy})(pq) + I_{xz}(qr) - I_{yz}(pr) + I_{xy}(q^2 - p^2) + I_1(w_1 q) + \\ - I_2(w_2 p) + mgr_x sin(\phi)\cos(\theta) + mgr_y sin(\theta) \\ p + \tan(\theta)[q sin(\phi) + r\cos(\phi)] \\ q\cos(\phi) - r sin(\phi) \\ \frac{1}{\cos(\theta)}[q sin(\phi) + r\cos(\phi)] \\ \frac{T_1}{I_1} \\ \frac{T_2}{I_2} \\ \frac{T_3}{I_3} \end{Bmatrix} \tag{8}$$

which in compact form is given by

$$[M]\{\dot{X}\} = \{f(X)\} \Rightarrow \{\dot{X}\} = [M]^{-1}\{f(X)\} \tag{9}$$

where M represent the mass matrix, X the state vector of the system and F(x) the function matrix given by the right hand side elements of Eq.(8)

3 Control law and parameters estimation

In order to design the control law, Eq.(9) needs to be linear. Therefore, assuming small angular displacements the equations of motion for the design purpose is

$$\begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} p \\ q \\ r \\ \phi \\ \theta \\ \psi \end{Bmatrix} + \begin{bmatrix} \frac{1}{I_1 - I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_2 - I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_3 - I_{zz}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} \quad (10)$$

which in state space form is given by

$$\begin{aligned} \dot{X} &= AX + Bu \\ Y &= CX \end{aligned} \quad (11)$$

where Y represents the system outputs (measures) and C the sensor location matrix.

A control law used is a PD type given by

$$\{u\} = -[K]\{X\} \quad (12)$$

where the gains are obtained by applying the pole allocation method (Ogata,1998).

The equations of motion obtained previouslly are in general form and can be applied for different satellite attitude system. The identification process employed uses the equations of motion in its linear matrix form. From now on the vector X consists of the parameters to be identified, here the elements of the inertia matrix and those terms give the location of the platform center of gravity. The elements of vector Y contain terms associated to the sensor measures and reaction wheels inertia matrix, therefore totally known.

The identification is done by applying an algorithm based on the least square method, in the form

$$[G]\{X\} = \{Y\} \quad (13)$$

where the matrix G contains the measurements of the angles, angular velocities and angular accelerations, which are functions of the types of sensor used. See (Souza, 2007) for details.
Considering that the matrixes $[G_K]$ and $\{Y_K\}$ are obtained in the instant (or step) k with measurements obtained at instant t(k-1), the recursive form of the least square method needs to satisfy the following equations

$$\begin{aligned} [L_K] &= [P_{K-1}][G_K]^T \left([I] + [G_K][P_{K-1}][G_K]^T\right)^{-1} \\ [P_K] &= ([I] - [L_K][G_K])[P_{K-1}] \\ \{X_K\} &= \{X_{K-1}\} + [L_K](\{Y_K\} - [G_K]\{X_{K-1}\}) \end{aligned} \quad (14)$$

where the initial values are given by

$$\begin{aligned} [P_0] &= \left([G_0][G_0]^T\right)^{-1} \\ \{X_0\} &= [P_0][G_0]^T\{Y_0\} \end{aligned} \quad (15)$$

It is observed that the estimation of $\{X_K\}$ is obtained by adding a correction to the estimation done previously in the step (k-1). The correction term is proportional to the difference between the measured values of Y_k and the measurements based on the estimation done in the previous step, given by $[G_K]\{\hat{X}_{K-1}\}$

The components of the vector $\{L_k\}$ are weight factors giving information about how the correction and the previous estimative should be combined. The matrix $[P_k]$ is only defined when the matrix $[\overline{G_K}]^T[\overline{G_K}]$ is not singular. To avoid singularities,the recursive process must be initiated with a big positive define matrix $[P_0]$.

4 Simulation results

4.1 Parameters estimation of the 3D simulator

The 3D simulator parameters estimation is done by simulating the dynamics given by Eq. (11) with the control law given by Eq. (12), using the platform and reaction wheel inertias and the external torque shown in Table 1 in international system unit.

TABLE 1. System data used in the estimation process.

Platform	Platform	Reaction wheel	External torque
I_{xx} = 1.1667	I_{xy} = 0.0107	I_1 = 0.001792	Mgr_x =0.0101
I_{yy} = 1.1671	I_{xz} = -0.0159	I_2 = 0.001792	Mgr_y =0.0323
I_{zz} = 2.1291	I_{yz} = 0.0159	I_2 = 0.001792	Mgr_z =0.7630

The parameters estimation obtained by applying the least square recursive method to the measurements are taken in time interval of 5s for a simulation period of 20s. Figures 3, 4 and 5 show the platform inertias and external torques parameters estimations, respectively.

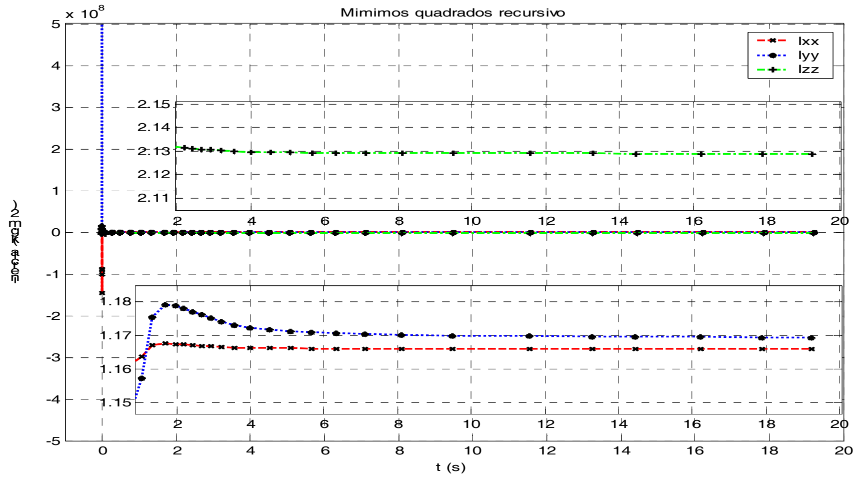

Figure 3. Platform principal inertia parameters estimation.

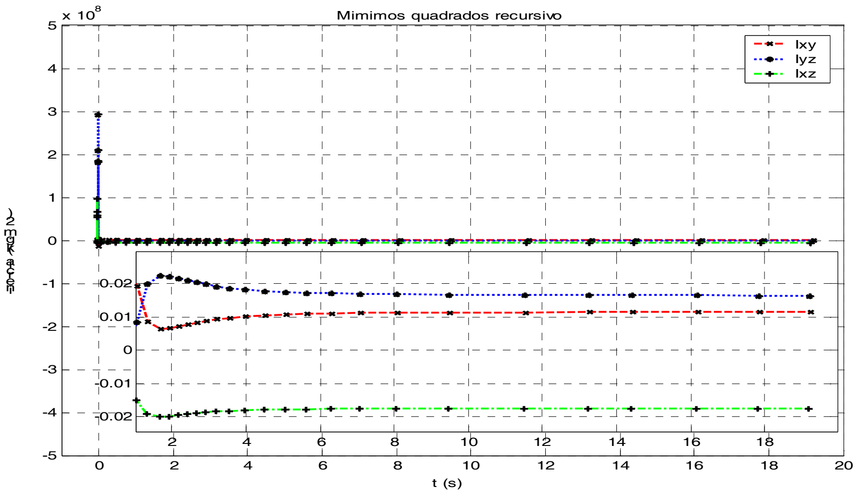

Figure 4. Platform cross inertia parameters estimation.

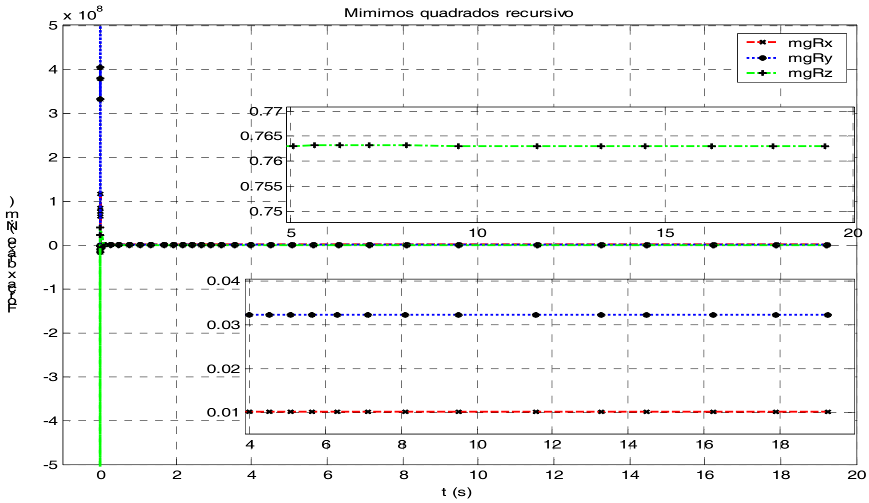

Figure 5. External torque parameters estimation.

From Figs. 3, 4, and 5 one observes that, although it is necessary a great number of interactions to obtain an accurately parameters estimative, the recursive procedure is stable, once the error tend to decrease with time. That behavior shows that the recursive least square method is adequate to estimate the system parameters.

4.2 Parameters estimation of 1D simulator

In order to apply the previous described recursive procedure to estimate the 1D simulator inertia moment using experimental data, the 3D equation of motion is simplified for rotation around the vertical axis. These simplification results in the following equations of motion

$$m|\vec{g}|r_y\vec{i} + 0\vec{j} - m|\vec{g}|r_x\vec{k} = \left(\dot{r}I_{xz} - r^2 I_{yz}\right)\vec{i} + \left(\dot{r}I_{yz} + r^2 I_{xz}\right)\vec{j} + \left(\dot{r}I_{zz} + \dot{w}_3 I_3\right)\vec{k} \qquad (16)$$

However, considering that the platform is rigidly attached to the base of the simulator, which means that the gravity and centrifugal forces are canceled by the platform support, so as Eq. (16) has a simpler linear form given by

$$[\dot{r}]\{I_{zz}\} = \{-I_3\dot{w}_3\} \qquad (17)$$

which can be included in Eq.(13) form and then it is possible to apply the recursive least square method given by Eq.(14). It is important to mention that now the gyroscope measures the platform angular velocity and the reaction wheel generates torque.

The information about the equipments used in the experiments is given in Table 1. Figure 6 shows the experiment set up with components, beside the battery, the antenna and interfaces used in the experiment that is described in the sequence.

Table 1. The equipments used to perform the experiments.

Sunspace reaction wheel	Sunspace Fiber Optics Gyroscope
Angular rotation -/+ 4200 rpm	Field of measure -/+ 80°/s
Maximum torque 50mNm	Freeware Radio-Modem 908 – 950 MHz
Maximum angular moment 0.65Nms	Rate: 110Kbps with RS-232 protocol
Inertia moment 1.5E-3 Kgm.m Battery 12Vdc Air baring platform diameters 650mm	National Instruments PC 1.26GHz Interface RS-232/RS-485 System Voltage : 12 Vdc

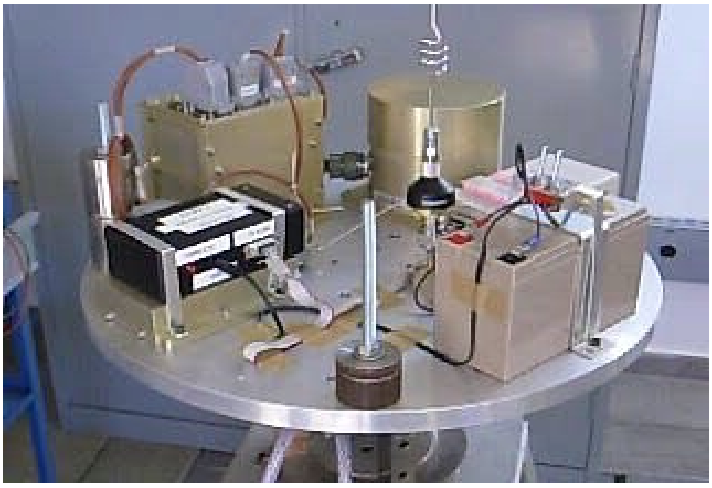

Figure 6. Simulator components used in the experiment.

In the experiment, initially, both platform angular and reaction wheel velocities are equal to zero. Then one sends a commander to the reaction wheel so that its angular velocity increases up to a certain value. After that, one sends another commander to decrease the reaction wheel angular velocity up to zero, as showed by the red line in Fig. 7. The reaction wheel acceleration, blue line, is obtained numerically

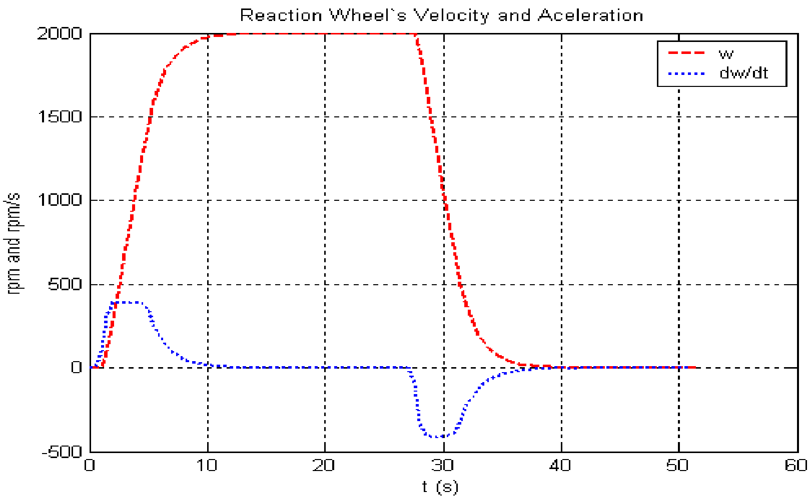

Figure 7. Reaction wheel velocity and acceleration.

That action makes the platform to rotate with opposite angular velocity, following a rotation other way around, as showed by the red line in Fig. 8. Platform acceleration, blue line, is also obtained numerically. During that process the gyroscope measures the platform angular velocity and the reaction wheel angular velocity is also measured.

It is important to say that the platform friction has been neglected. The value of inertia moment estimated is around $0.51Kgm^2$. See Fig. 9.

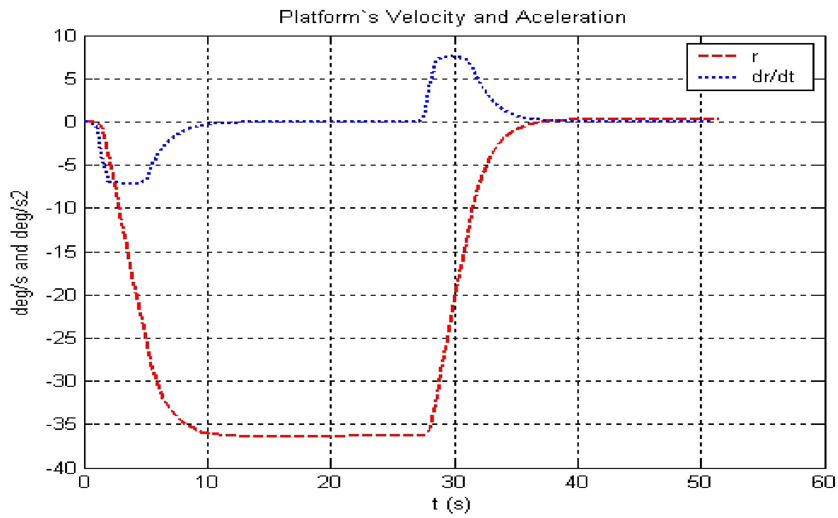

Figure 8. Platform angular velocity and acceleration.

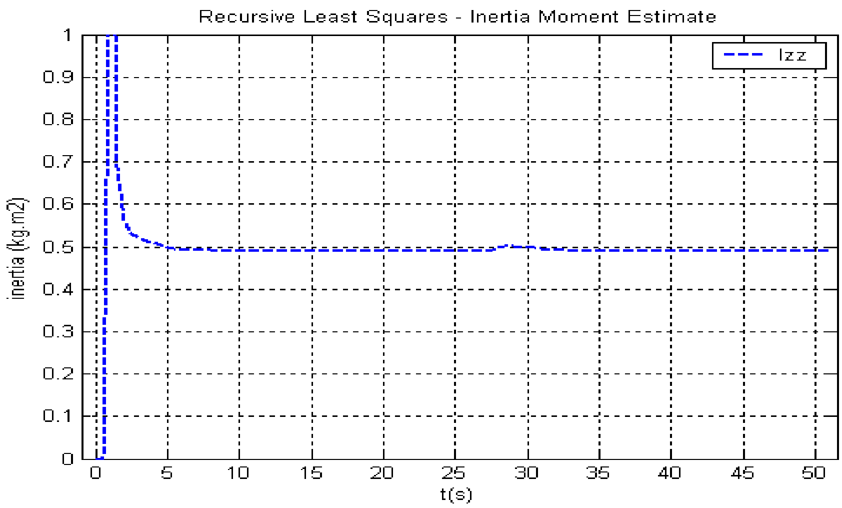

Figure 9. Platform inertia moment estimated.

5 Conclusions

In this paper a 3D mathematical model of a platform that permit to simulate a satellite ACS is developed. The model is represented by the satellite equations of motion in three axes, with three reactions wheel as actuators and three gyroscopes as angular sensors. The control system is designed based on PD control laws using the poles allocation method. That simulator model is used to generate data to estimate the inertia parameters of the 3D platform by the least square recursive method. The simulations have shown that the recursive algorithm is reliable. The 3D estimation algorithm is simplified for 1D platform prototype from which real data is obtained to estimate the 1D platform inertia moment with great precision. It is important to stress that the inertia moment estimated by this procedure is in agreement with the values obtained by other method (Carrara and Milani, 2007).

6 References

Ahmed, J. Coppola V. T. e Bernstein D., “Adaptative asymptotic tracking of spacecraft attitude motion with inertia matrix identification’, Journal of Guidance, Control and Dynamics, Vol. 21, pp. 684-691, 1998.

Berry, M. M., Naasz, B. J., Kim, H.Y. e Hall, C. D., “Integrated Orbit and Attitude Control for a Nanosatellite with Power Constrains”. AAS/AIAA Space Flight Mech. Conf, P. Rico, Fevereiro, 9-12, 2003.

Bergmann, E. V. e Dzielski, J., “Spacecraft mass property identification with torque-generating control”, Journal of Guidance, Control and Dynamics, Vol. 13, Jan-Feb., p. 99-103. 1990.

Cannon, R. H. e Rosenthal, D.E.,”Experiments in control of flexible structures with noncolocated sensors and actuators”, Journal of Guidance. AIAA, 7(5), 546-553, 1984.

Carrara, V. and Milani, P. G., “Controle de uma Mesa de Mancal a Ar de um Eixo Equipado com Giroscópio e Roda de Reação”. V Simpósio de Brasileiro de Engenharia Inercial, 26 a 27 de novembro de 2007. Rio de Janeiro, Brasil.

Dichmann, D. e Sedlak, J., "Test of a Flexible Spacecraft Dynamics Simulator". Published by AAS in Advances in the Astronautical Sciences, Vol. 100 Part I, pp. 501-526, paper AAS 98-340, Ed. Thomas H. Stengle, USA, 1998.

Hall, C.D., Tsiotras and Shen, H. “Tracking Rigid Body Motion Using Thrusters and Momentum Wheels”, Journal of the Astronautical Sciences, 2002.

Kaplan. M. H, Modern Spacecraft Dynamics and Control, John Wiley & Sons New Your, 1976.

Lee, A. Y. e Wertz, J. A., “In-flight estimation of the Cassini spacecraft inertia tensor”, J. Spacecraft, vol. 39, no. 1, pp. 153-155, 2002.

Ogata, Katsushiko, “Modern Control Engineering”, Prentice Hall, 3rd edition. 1998.

Peterson, R.L., “Air-Bering Spin Facility for Measuring Energy Dissipation”. NASA TN-D-8346, Outubro, 1976.

Peterson, R.L., “Air-Bering Spin Facility for Measuring Energy Dissipation”. NASA TN-D-8346, Outubro, 1976.

Souza, L. C. G., “Dynamics and Robust Control for Uncertain Flexible Space System”, PhD Thesis, Cranfield Institute of Technology, CoA, Cranfield, England, 1992.

Souza, L. C. G., “Modeling a Satellite Control System Simulator”, In the Proceeding of the AAS/AIAA Astrodynamics Specialist Conference 19-23 August - Mackinac Island, Michigan, USA. 2007.

Tanygin, S. e Williams, T., “Mass property estimation using coasting maneuvers”, Journal of Guidance, Control, and Dynamics, Vol. 11, May-June, p. 625-632. 1997.