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Spectral Moment Estimation and Fitting

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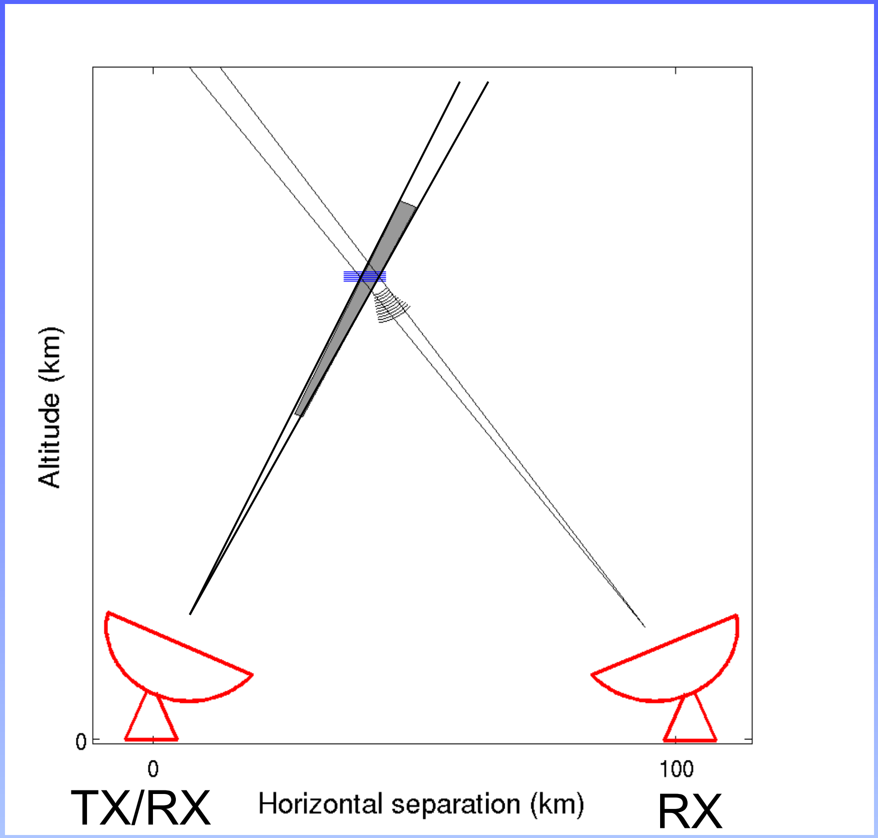
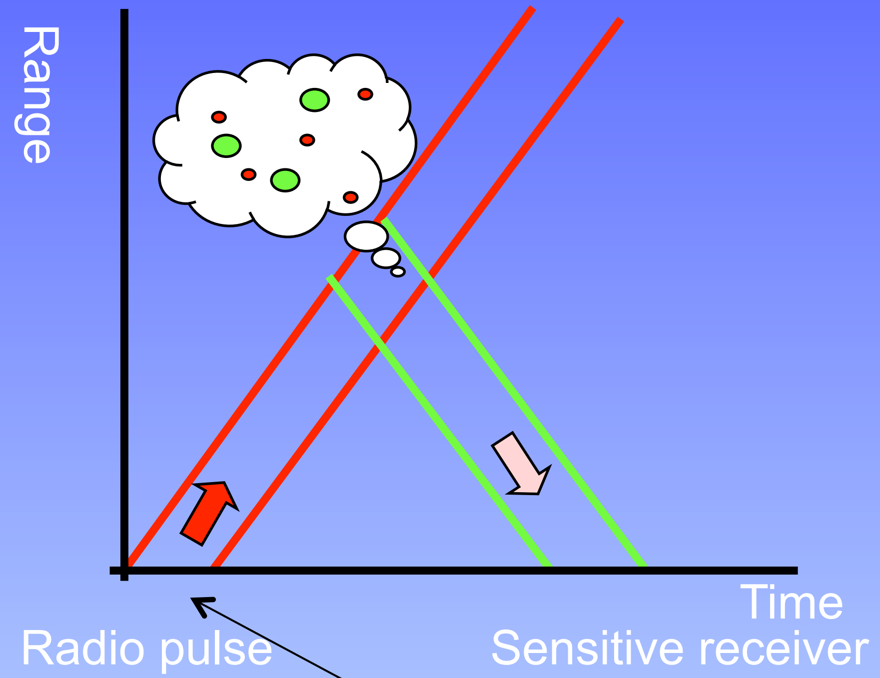
Radar Basics



Monostatic Signal



Significant improvements from pulse coding!

$$P_r = P_t \frac{G_t(\theta,\phi)}{4\pi R^2} \sigma(\omega,\vec{k},\chi) \frac{1}{4\pi R^2} A_{eff}(\theta,\phi)$$

Ambiguity Functions

- A standard way to compare different pulse coding strategies
- Based on the principle of a 'matched filter'
  - Output of the matched filter maximizes the attainable SNR when both signal and white noise are applied to the input
  - Impulse response of the matched filter is the complex conjugate of the time-reversed version of the signal

$$h(t) = s^*(t_M - t)$$

$$H(f) = S^*(f)\exp(-j2\pi f t_M)$$

where

$h(t)$ is the impulse response of the matched filter

$s(t)$ is the signal to be detected

t_M is the measurement time

t, f are time and frequency

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Ambiguity Functions

The ambiguity function is defined as the absolute value of the envelope of the output of a matched filter when the input to the filter is a Doppler shifted version of the original signal

$$\left|X(\tau, f)\right| = \left|\int_{-\infty}^{\infty} u(t)u^*(t-\tau)\exp(j2\pi ft)dt\right|$$

$u(t)$ is the complex envelope of the signal

τ is the additional delay

f is the frequency shift (Doppler)

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Ambiguity Functions

For $u(t)$ with unit energy

$$\left|X(\tau, f)\right| \le \left|X(0,0)\right| = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|X(\tau, f)\right|^2 d\tau df = 1$$

and for all signals

$$\left|X(-\tau,-f)\right| = \left|X(\tau, f)\right|$$

if $u(t) \leftrightarrow \left|X(\tau, f)\right|$

then $u(t)\exp\left(j\pi kt^2\right) \leftrightarrow \left|X(\tau, f + k\tau)\right|$

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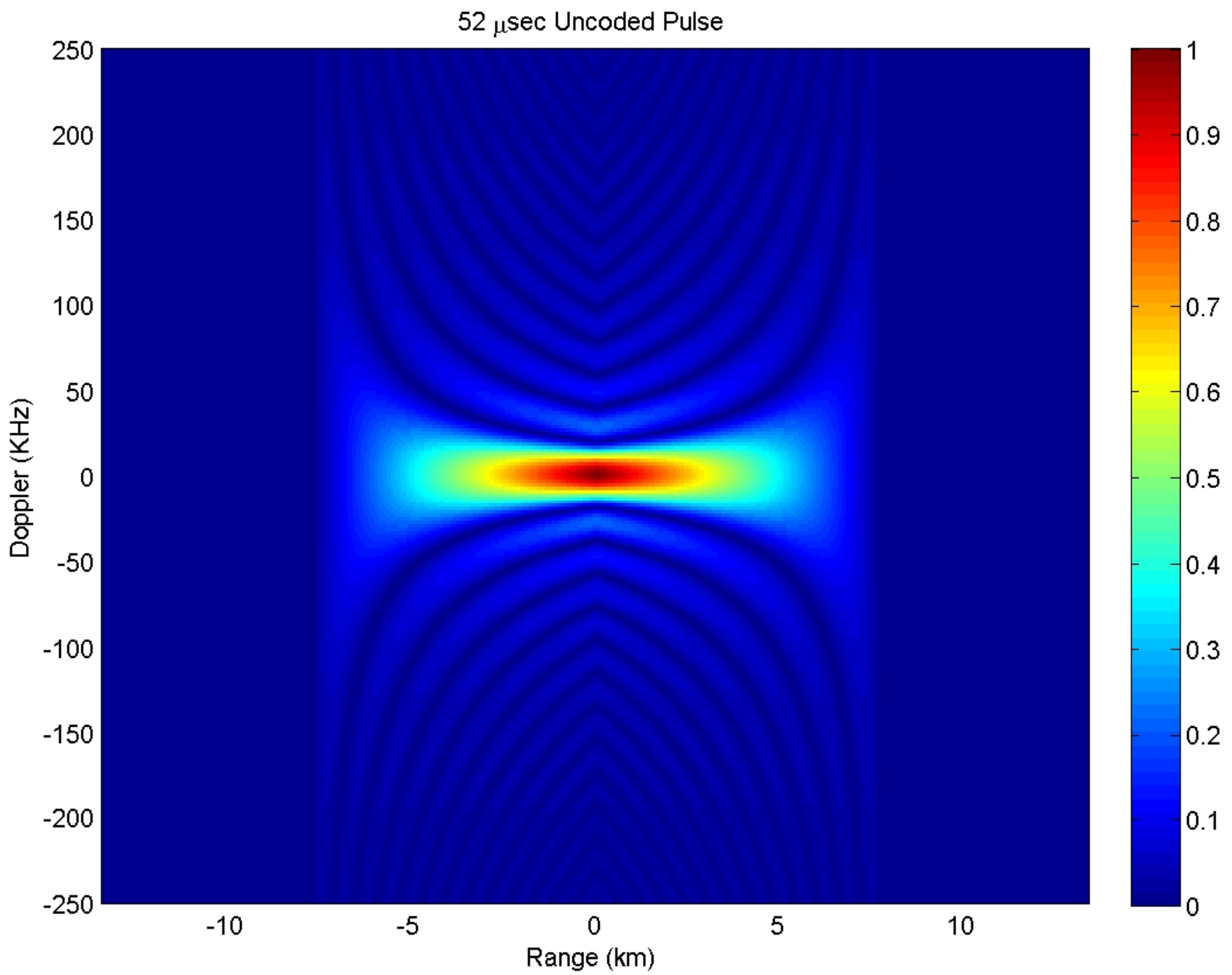
Ambiguity Function



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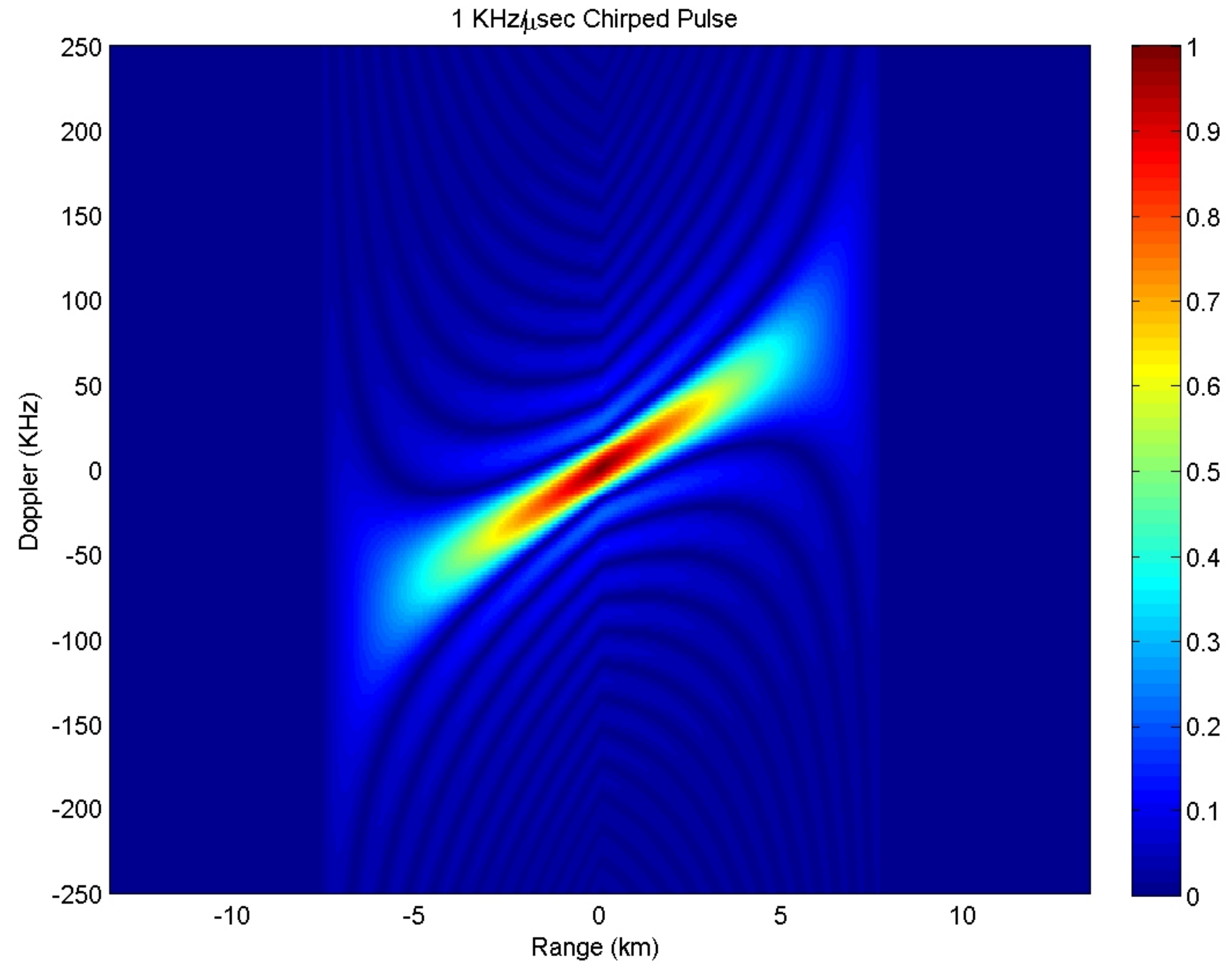
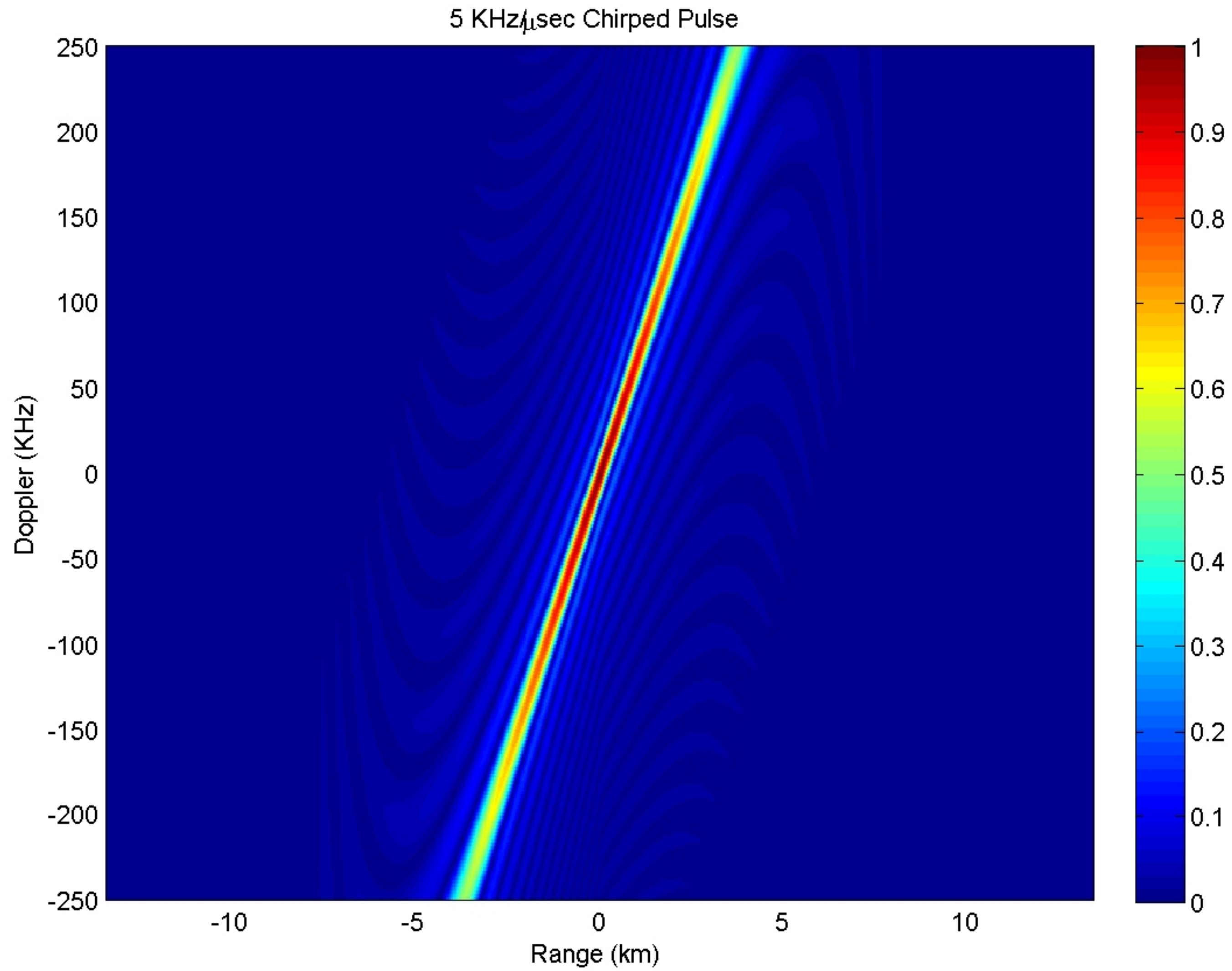
Ambiguity Function



Ambiguity Function



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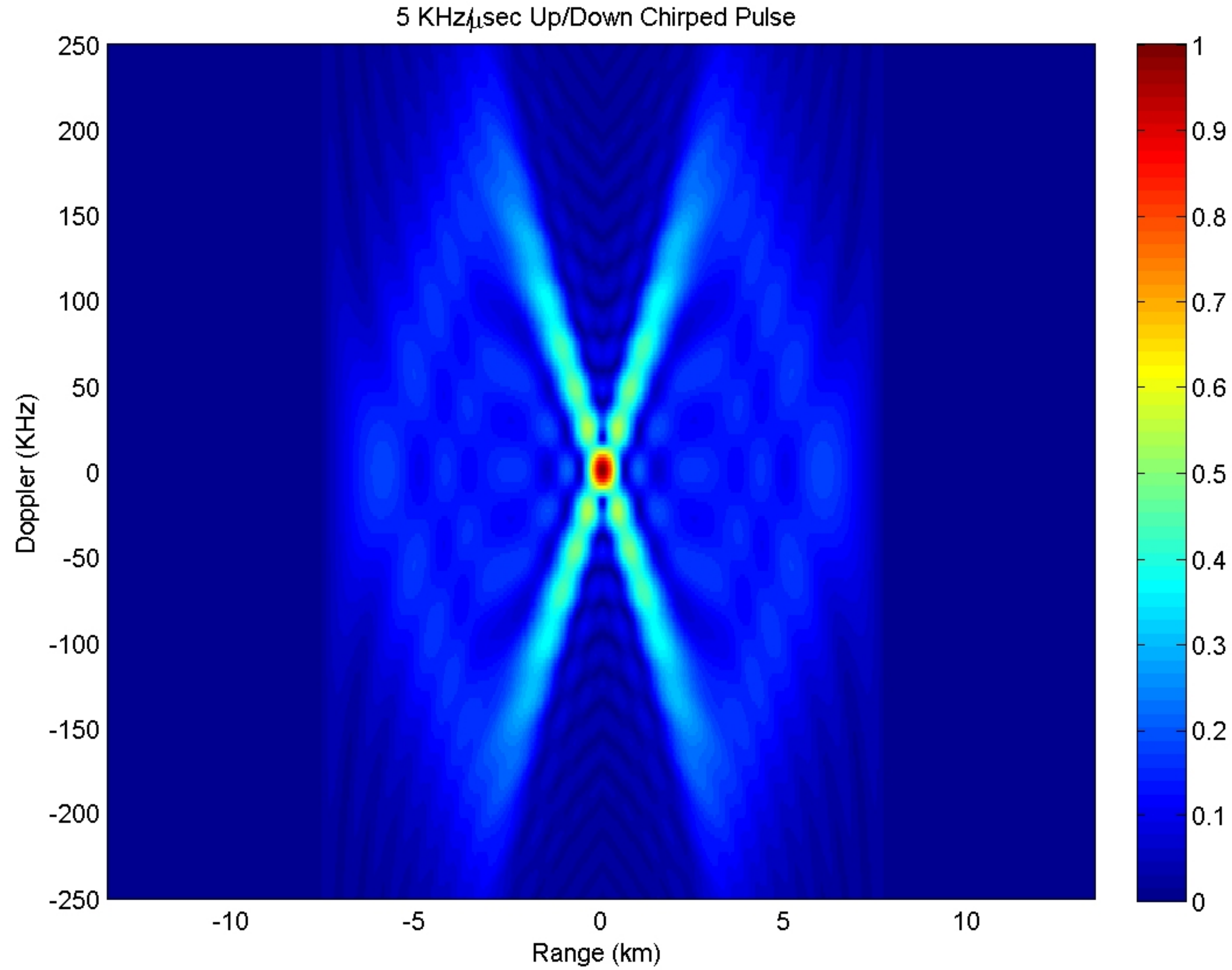
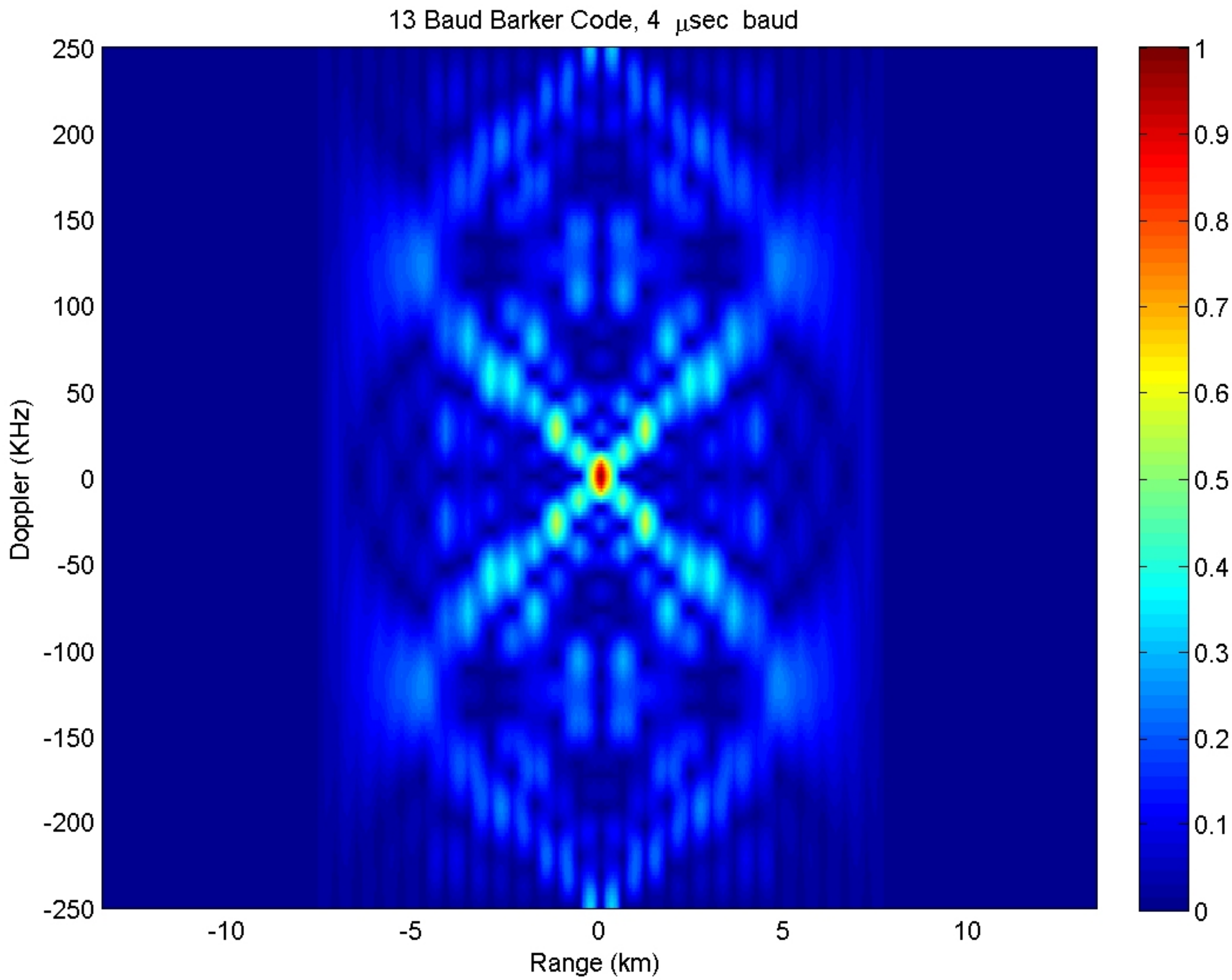
Ambiguity Function



Ambiguity Function



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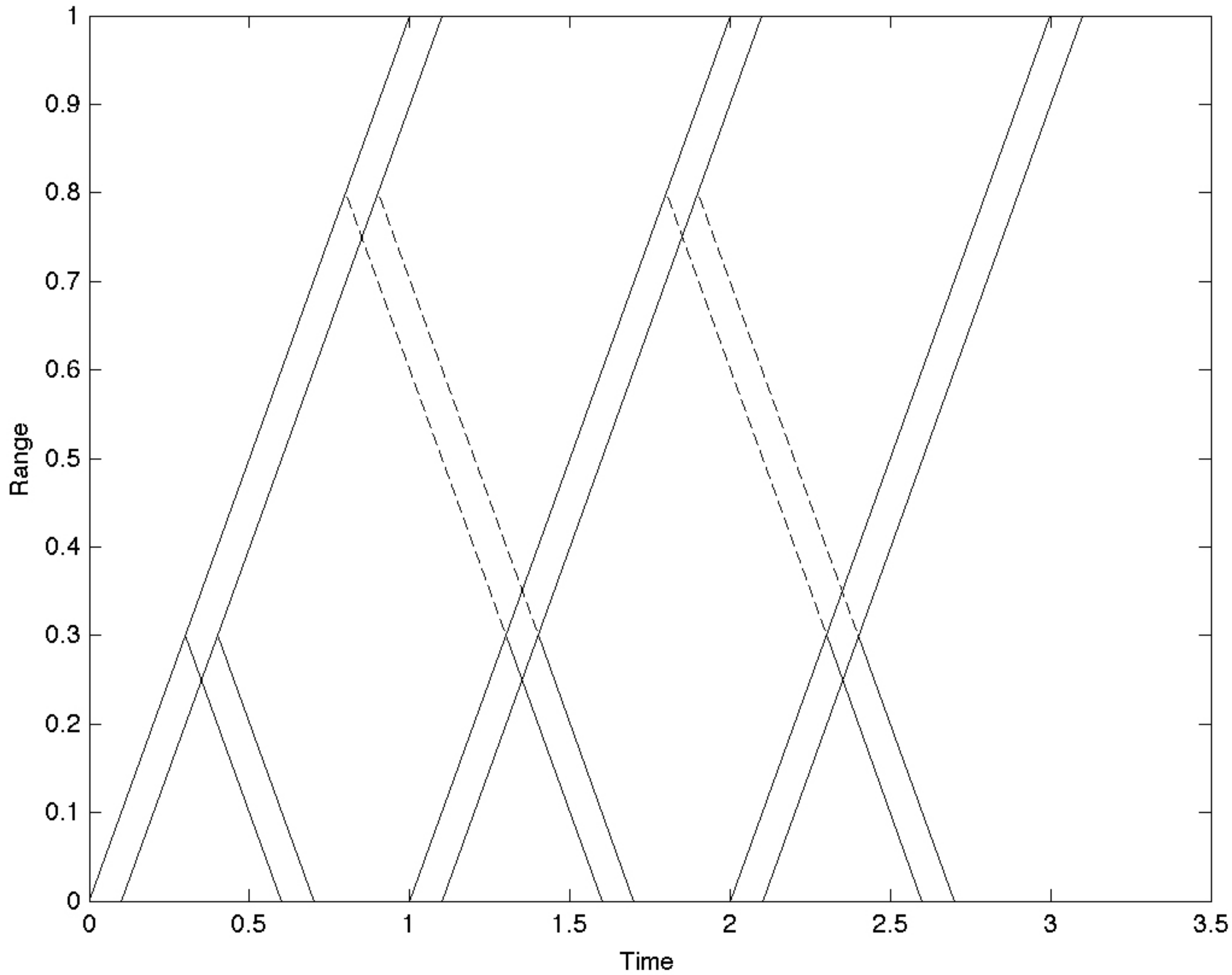
Pulse-to-Pulse Processing



“Typical” Radar Target

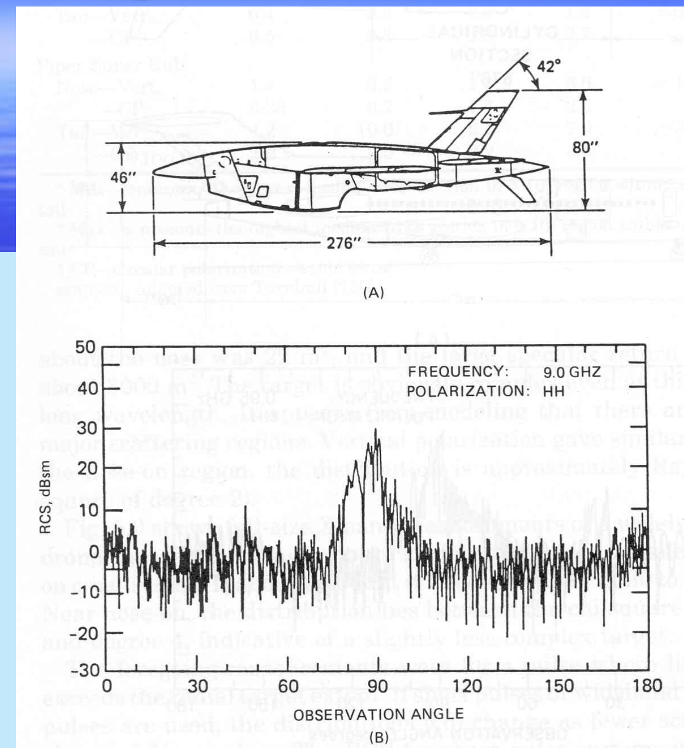
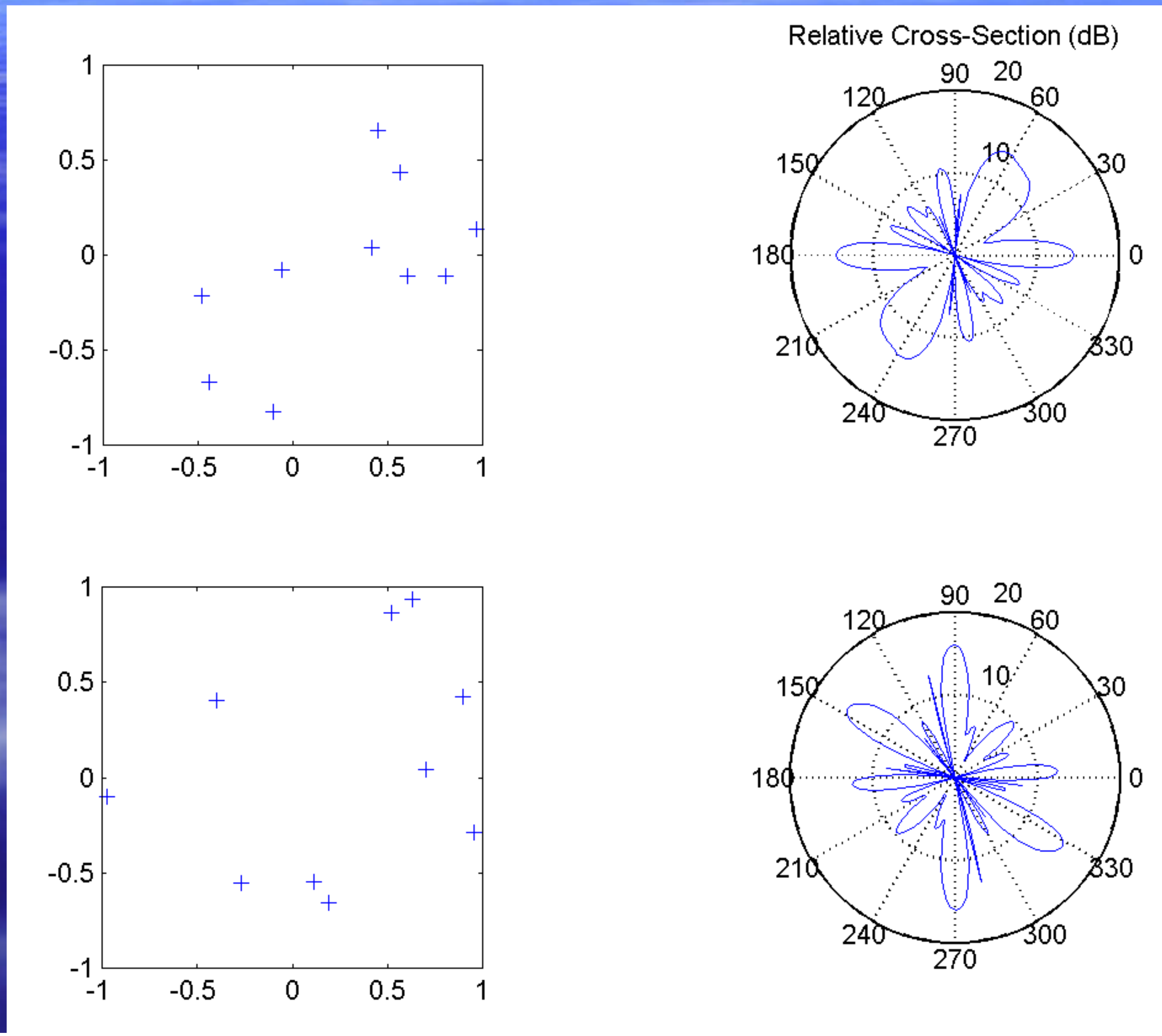


Figure 5.9 Teledyne Ryan drone. (A) Drawing; (B) measured RCS. (*From Follin et al.* [241])

Distributed Radar Target

Volume (Incoherent) Scattering

noise-like signal



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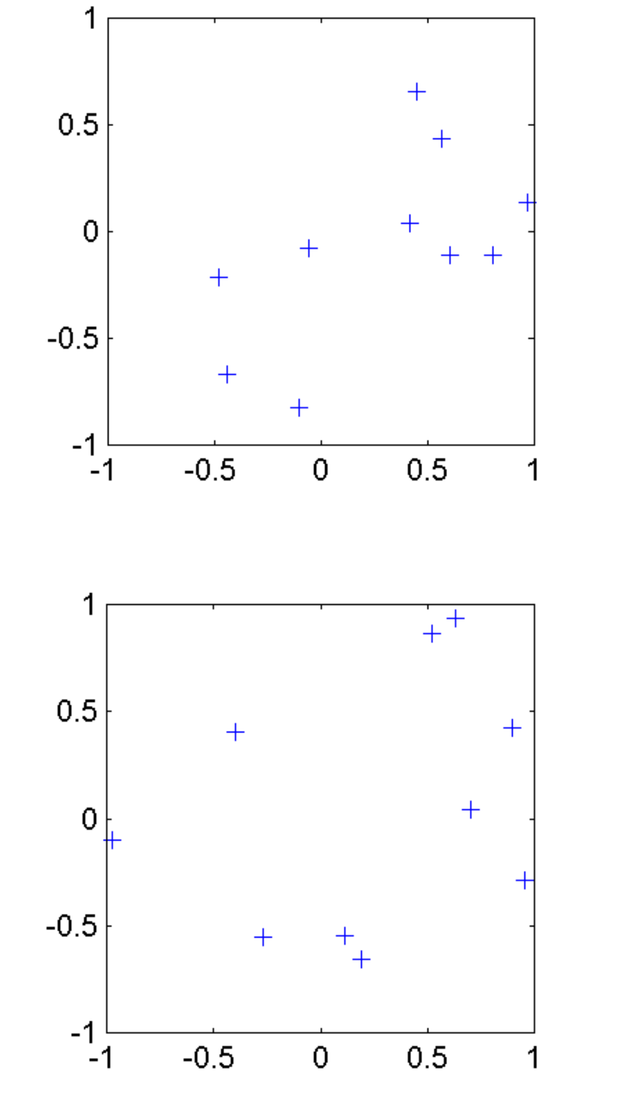
Bragg Scattering

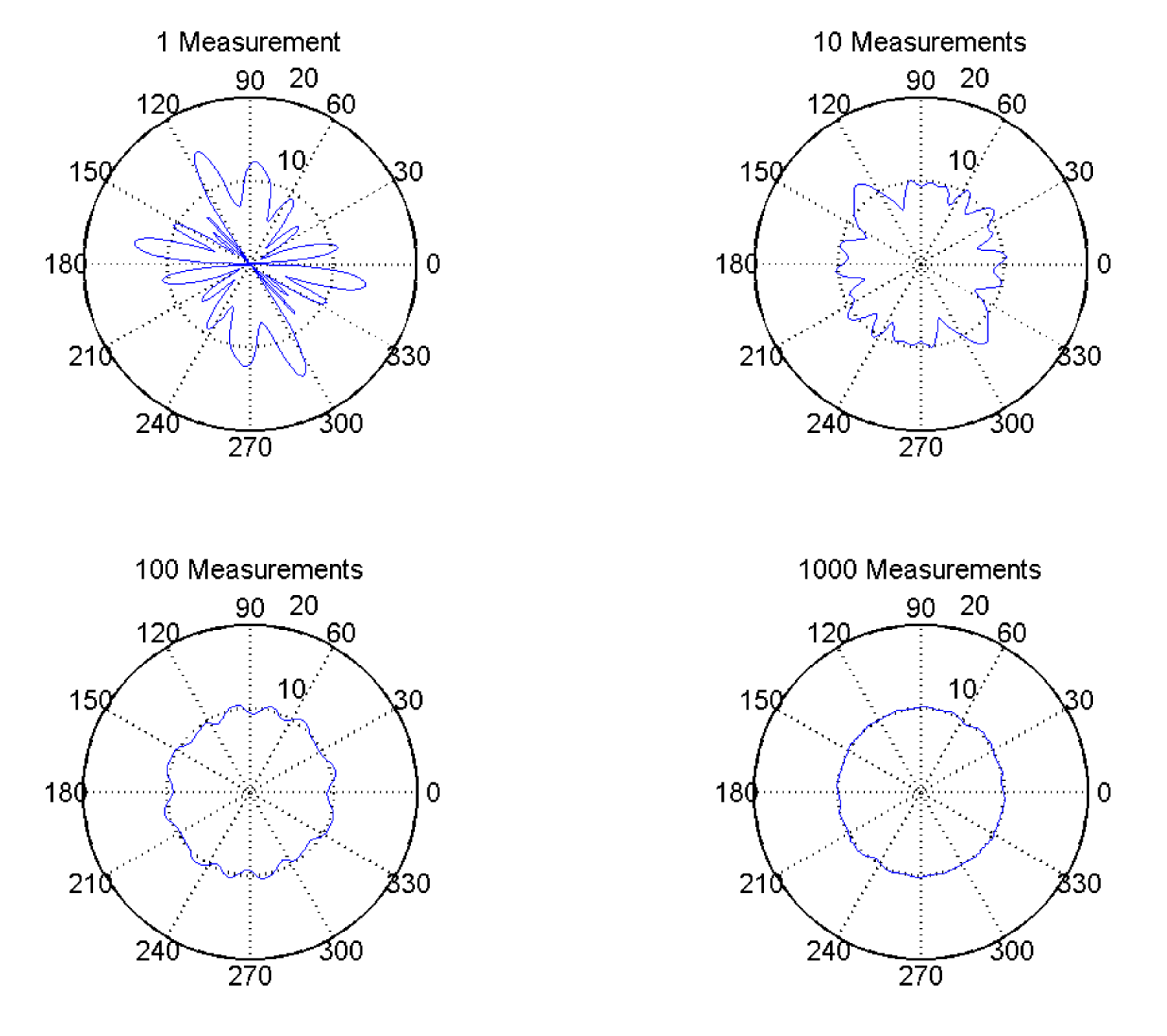
The radar, due to it's transmitted frequency/ wavelength, is picking out one component of a spatial fourier transform of the effective scattering centers (single k-vector).

Which component is selected is affected by both the transmitter and the scattering geometry (e.g. bistatic scattering angle), k_{rx}-k_{tx} = $2k_{rx}$ for backscatter

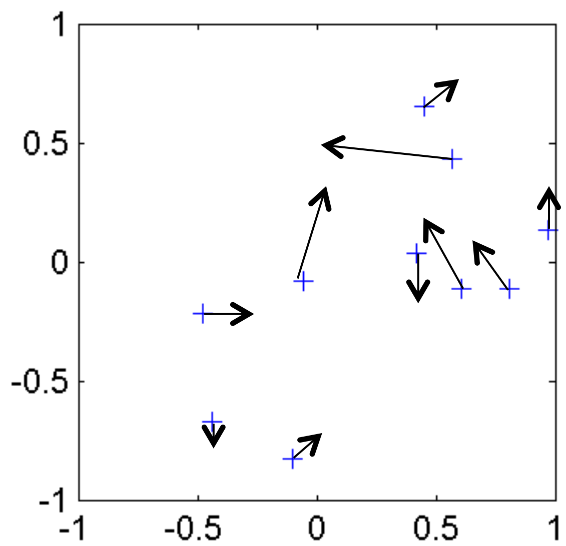
Use of multiple radars simultaneously can, thus, provide great insight into the physics of the medium by probing different spatial fourier transform components.

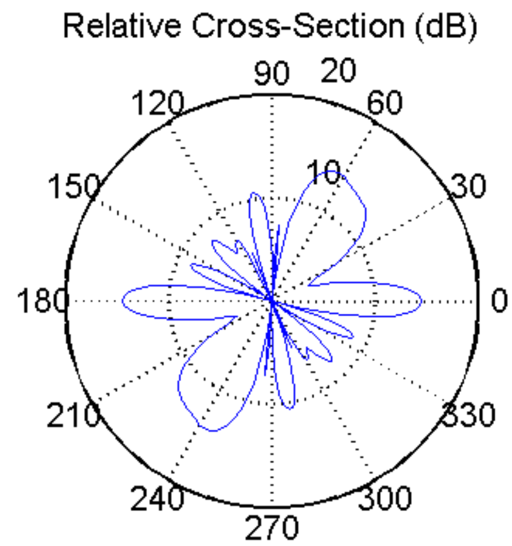


‘Incoherent’ Averaging



Spectral Measurements?





Weiner-Khinchin Theorem:

For a process that is at least wide-sense stationary ($ACF(t+\tau,t)=ACF(\tau)$), the autocorrelation function ($ACF(\tau)$) and power spectral density ($PSD(f)$) form a Fourier transform pair.

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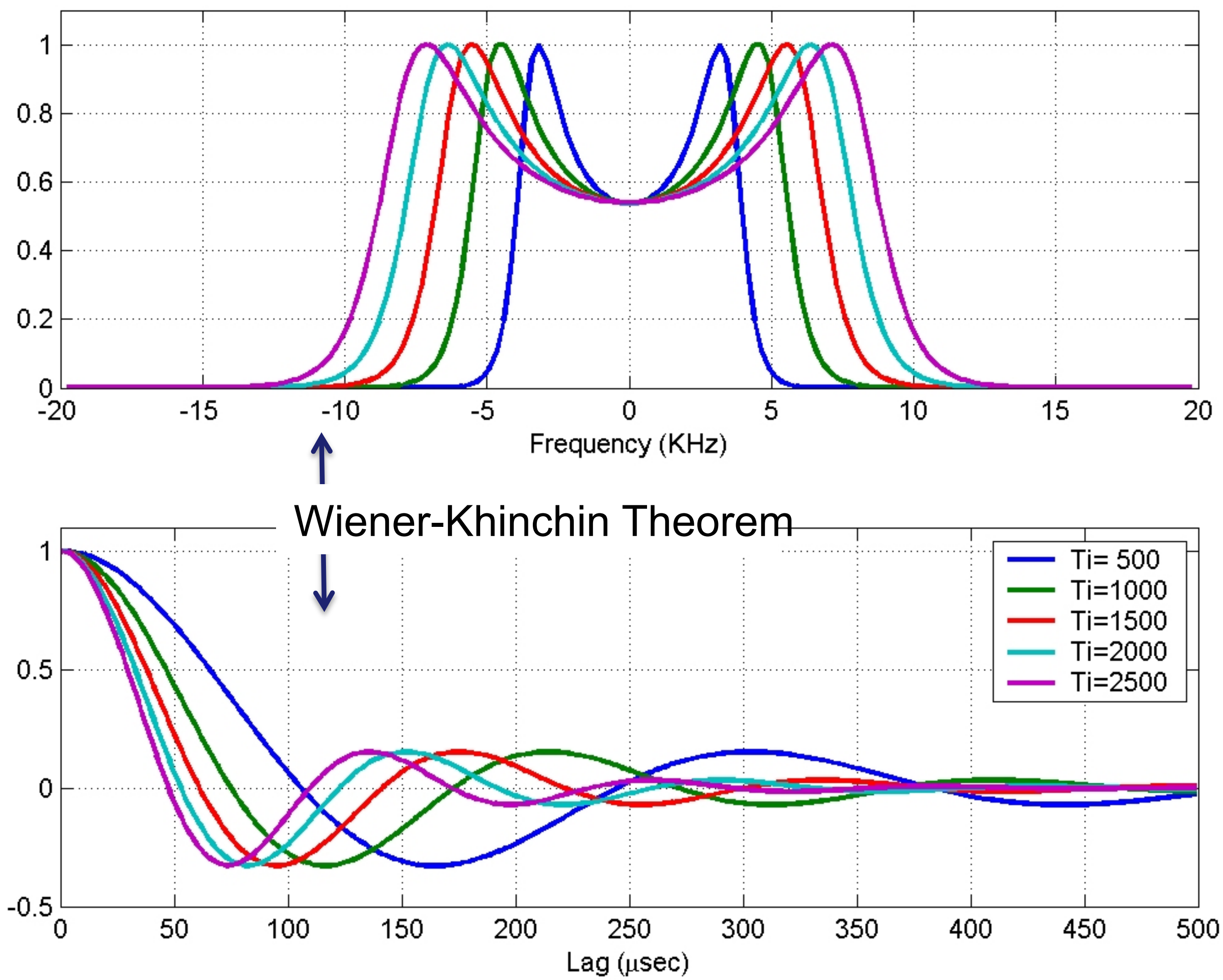
Wiener-Khinchin Theorem



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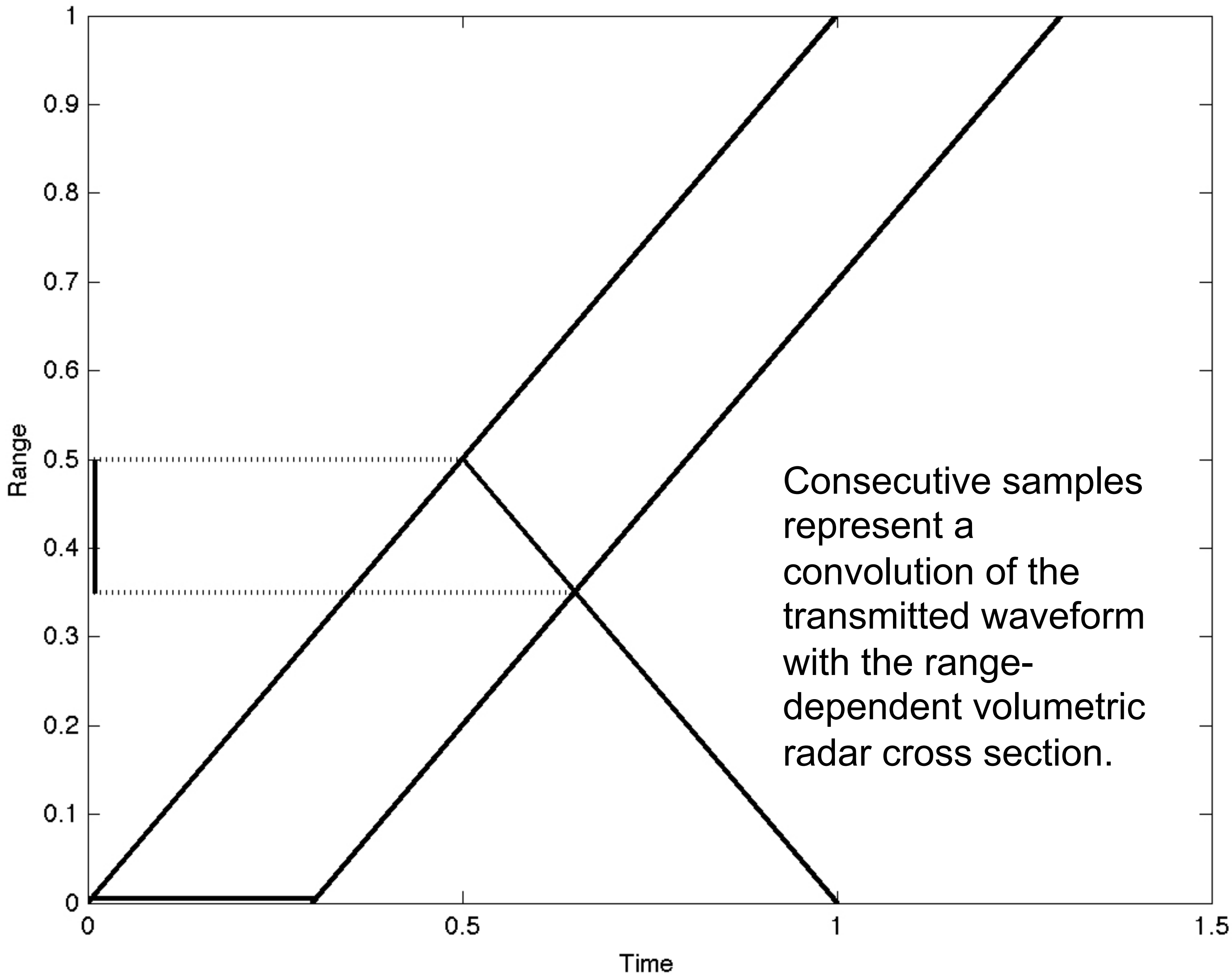
Range Smearing



Consecutive samples represent a convolution of the transmitted waveform with the range-dependent volumetric radar cross section.

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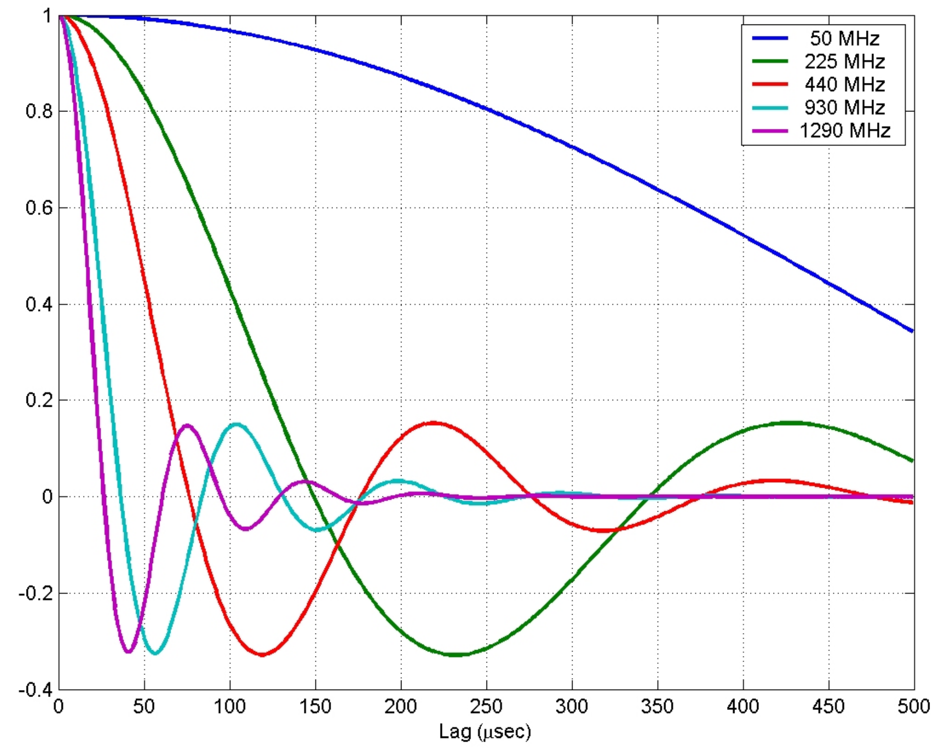
Can we still use phase coding?

Yes, but we must be careful!

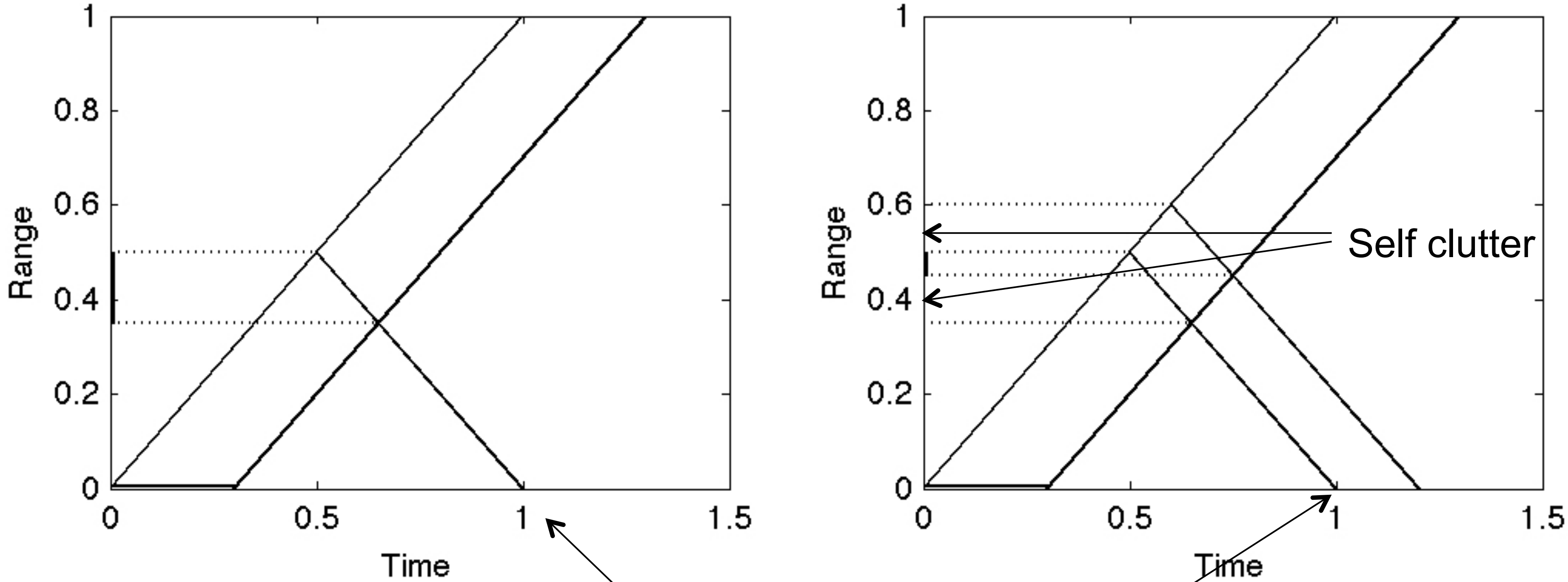
Barker codes, for instance, can be used if the total code length is sufficiently short (less than the correlation time of the medium – Gray and Farley, 1973). This only gives us power (0-lag) information!

Other classes of modulation are also available that, when incoherently averaged, provide good range resolution at the expense (usually) of increased bandwidth and processing complexity

- Alternating Codes (Lehtinen and Haggstrom, 1987)
- Coded Long Pulse (Sulzer, 1986)
- Compressed Alternating Codes
- Multipulse
- Complementary codes



Measuring ACFs



Receiver filter smears the time of the samples

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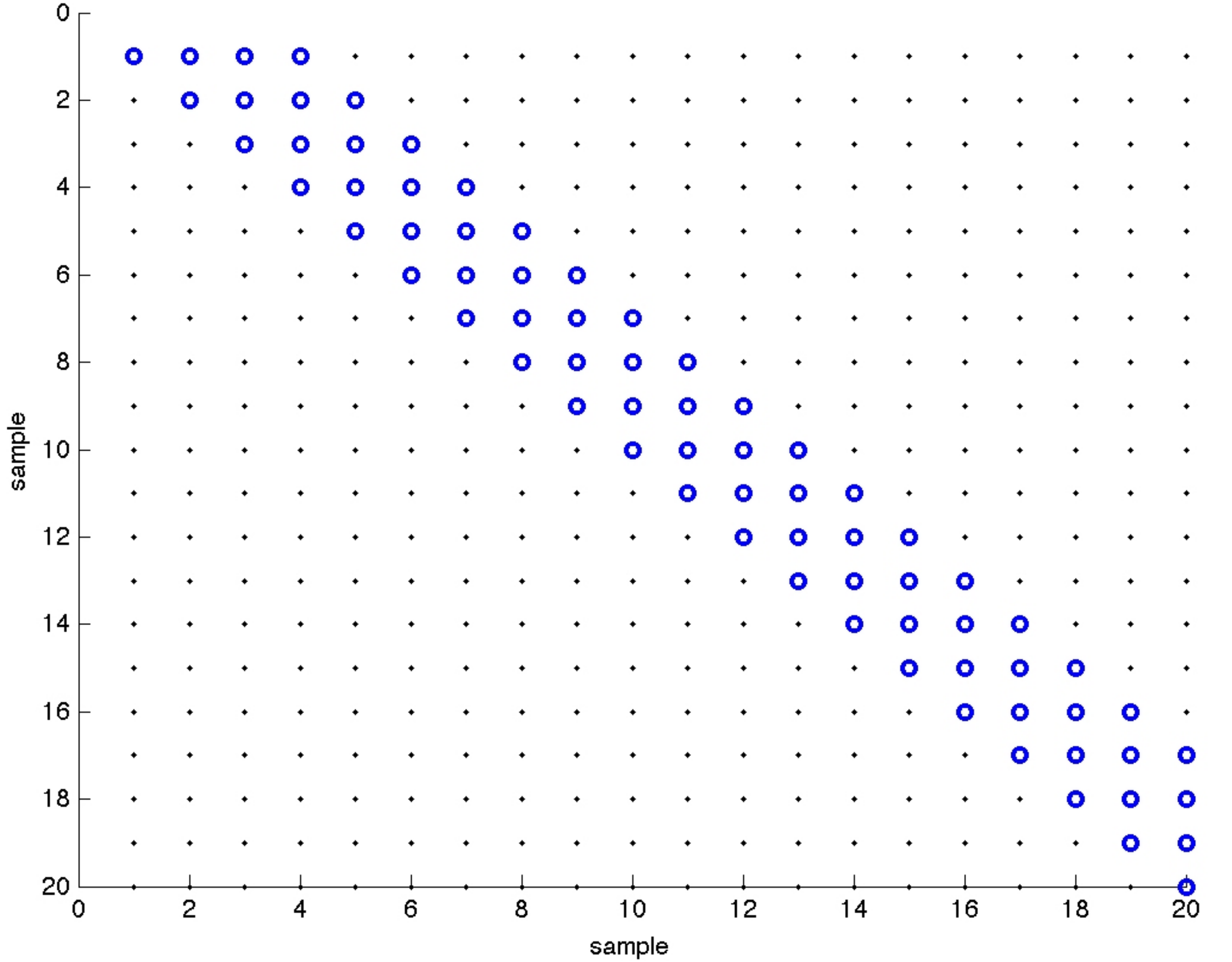
Lag Profile Matrix



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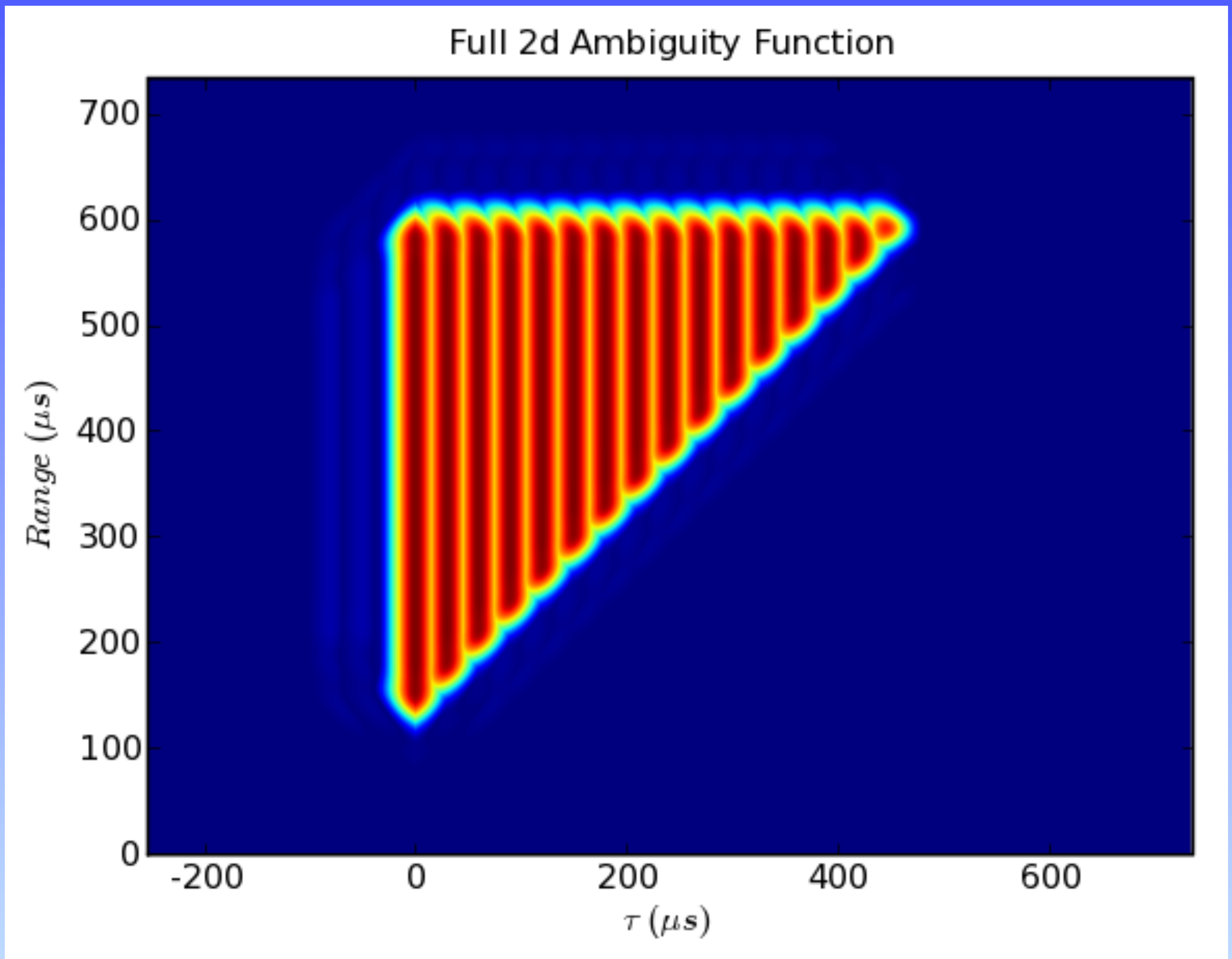
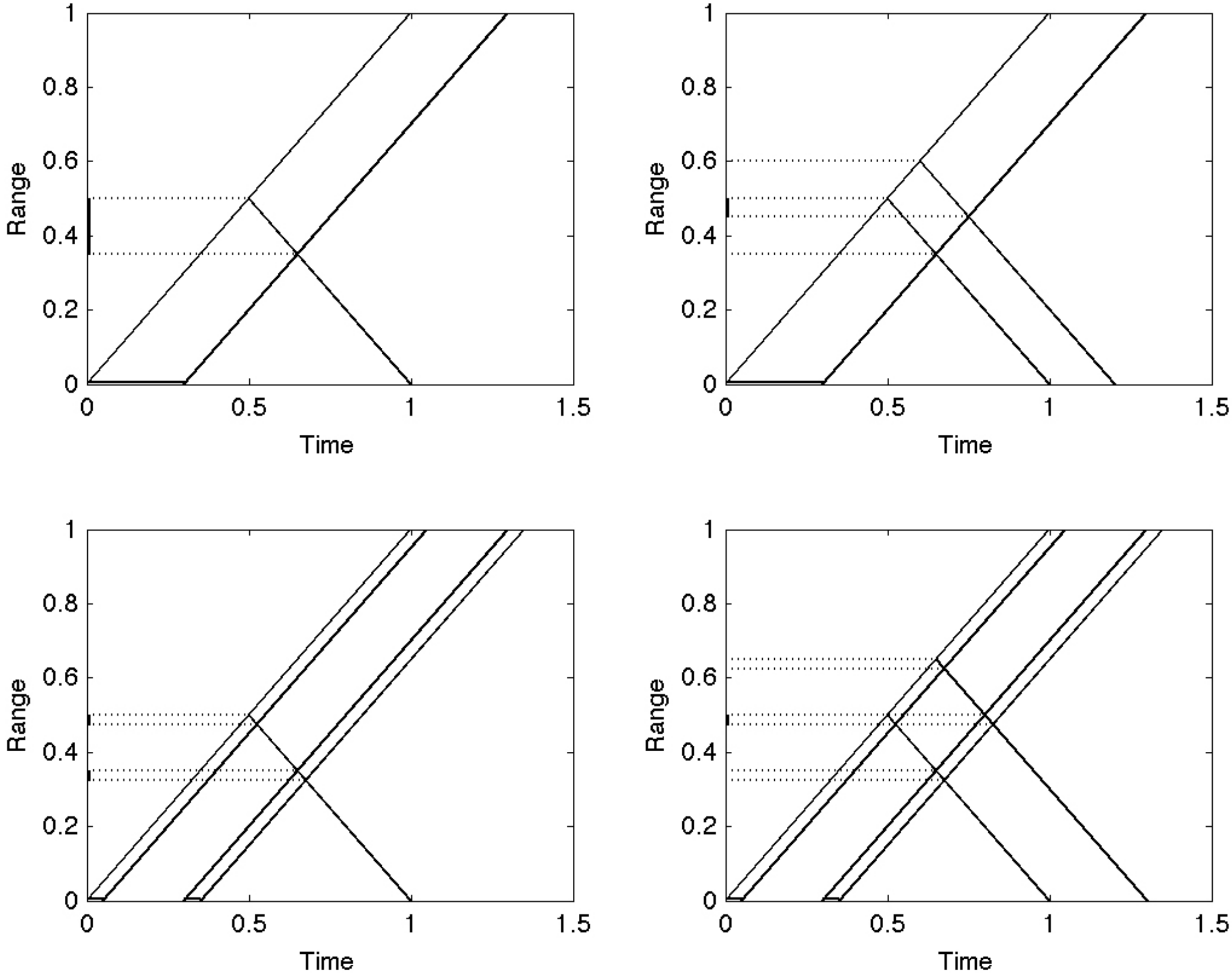
Ambiguity Function (smearing in range and lag)



Range Smearing



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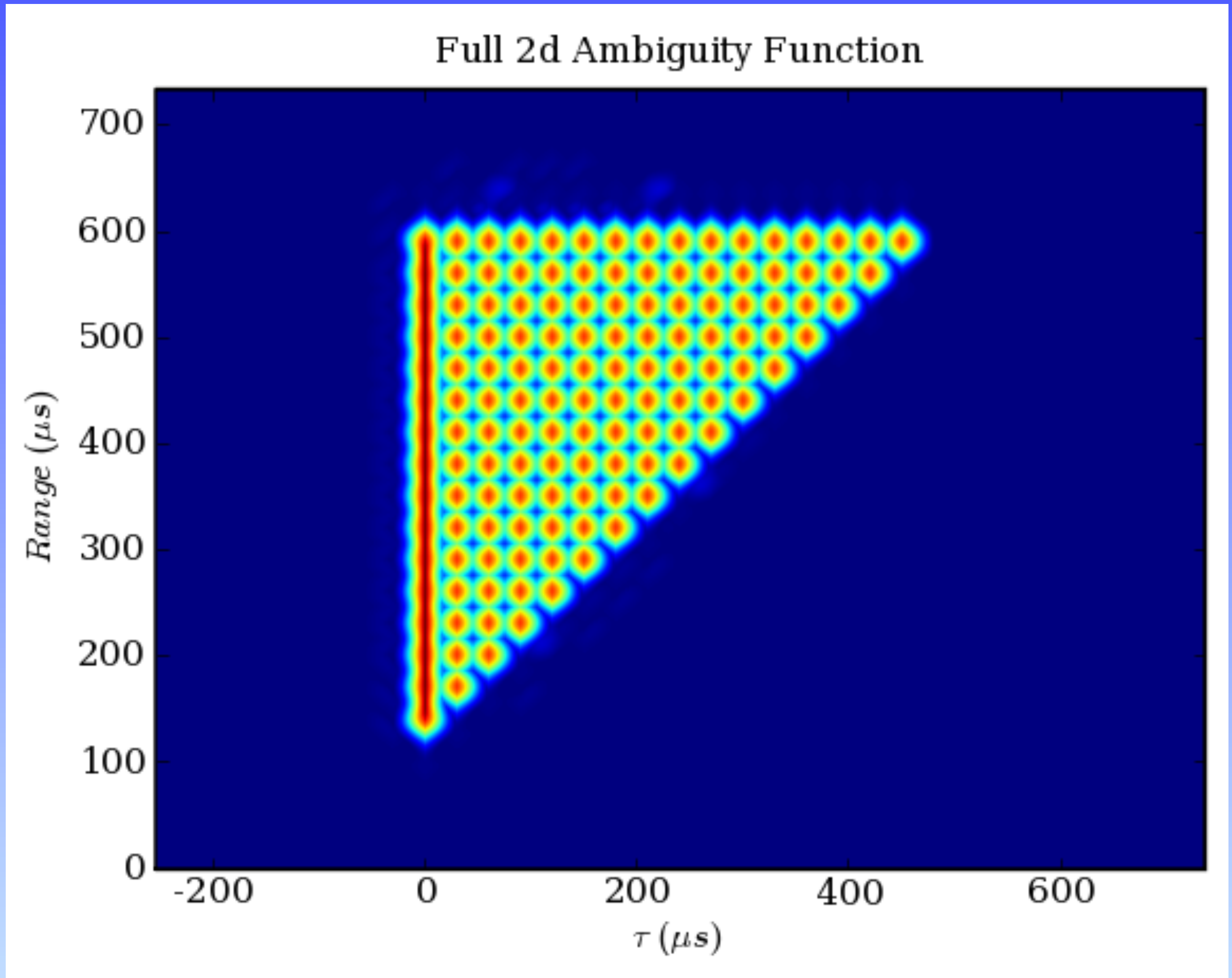
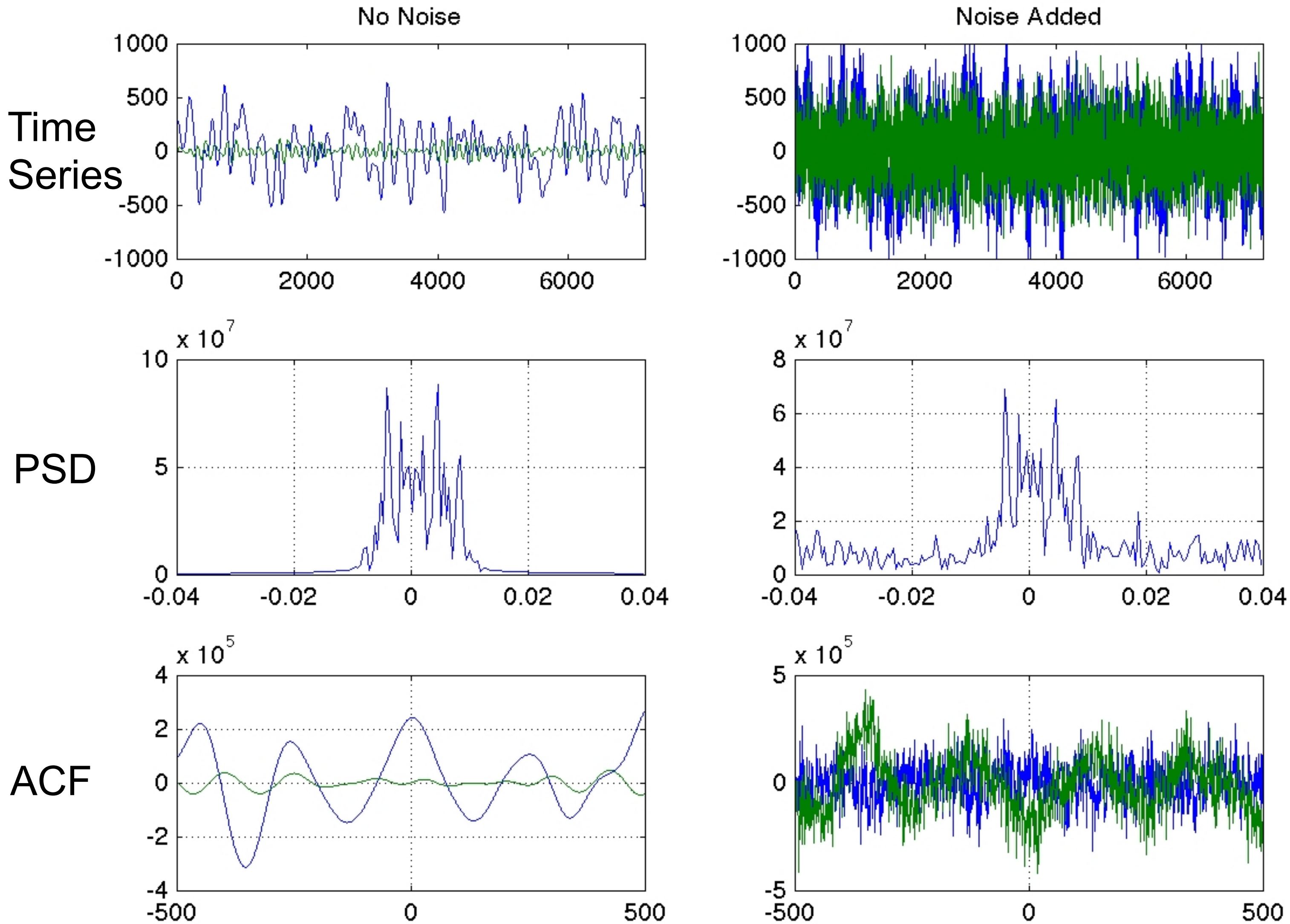
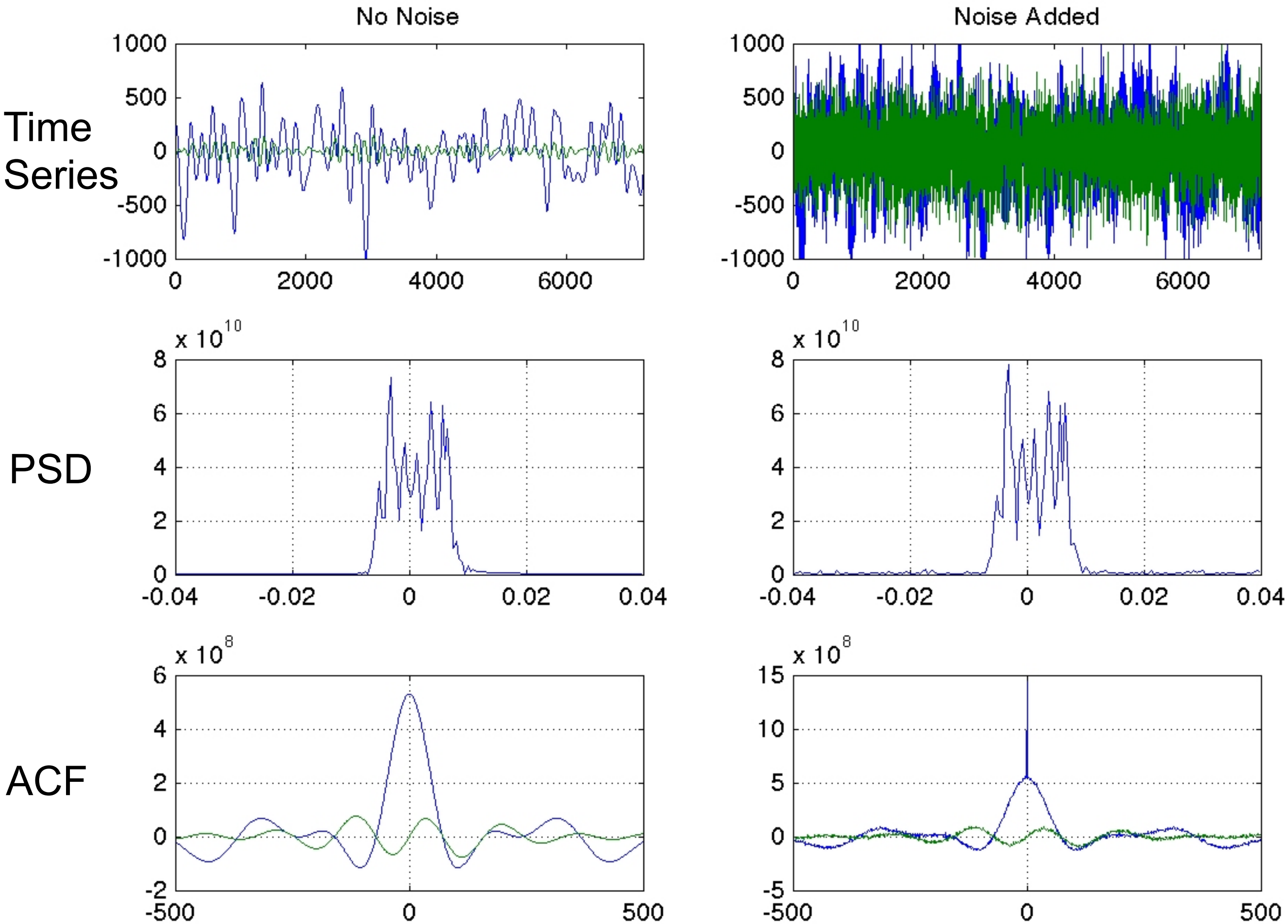
Alternating Code (smearing in range and lag)



Noise-like Signals + Noise



Noise-like Signals + Noise



Bulk Doppler shift in PSD => phase chirp in ACF

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Fourier Transform Properties

Operation	Time Function	Fourier Transform
Linearity	$af_1(t) + bf_2(t)$	$aF_1(\omega) + bF_2(\omega)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Time scaling	$f(at)$	$\frac{1}{\|a\|} F\left(\frac{\omega}{a}\right)$
Time transformation	$f(at - t_0)$	$\frac{1}{\|a\|} F\left(\frac{\omega}{a}\right)e^{-j\omega t_0/a}$
Duality	$F(t)$	$2\pi f(-\omega)$
Frequency shift	$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
Convolution	$f_1(t)*f_2(t)$	$F_1(\omega)F_2(\omega)$
	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega)*F_2(\omega)$
Differentiation	$\frac{d^n[f(t)]}{dt^n}$	$(j\omega)^n F(\omega)$
	$(-jt)^n f(t)$	$\frac{d^n[F(\omega)]}{d\omega^n}$
Integration	$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$

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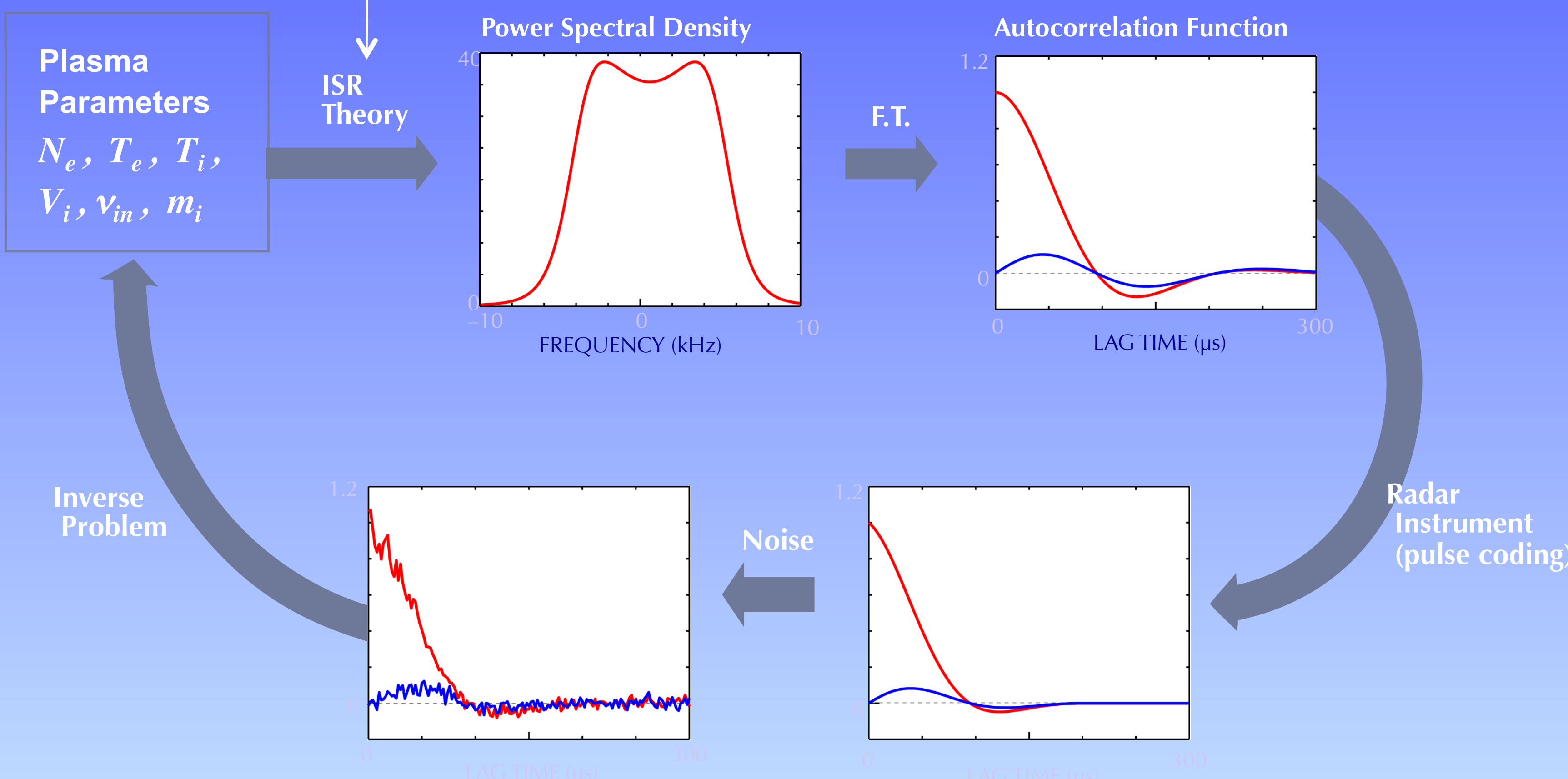
Incoherent Scatter Radar

This only works if different physical parameters impact the spectral shape differently!



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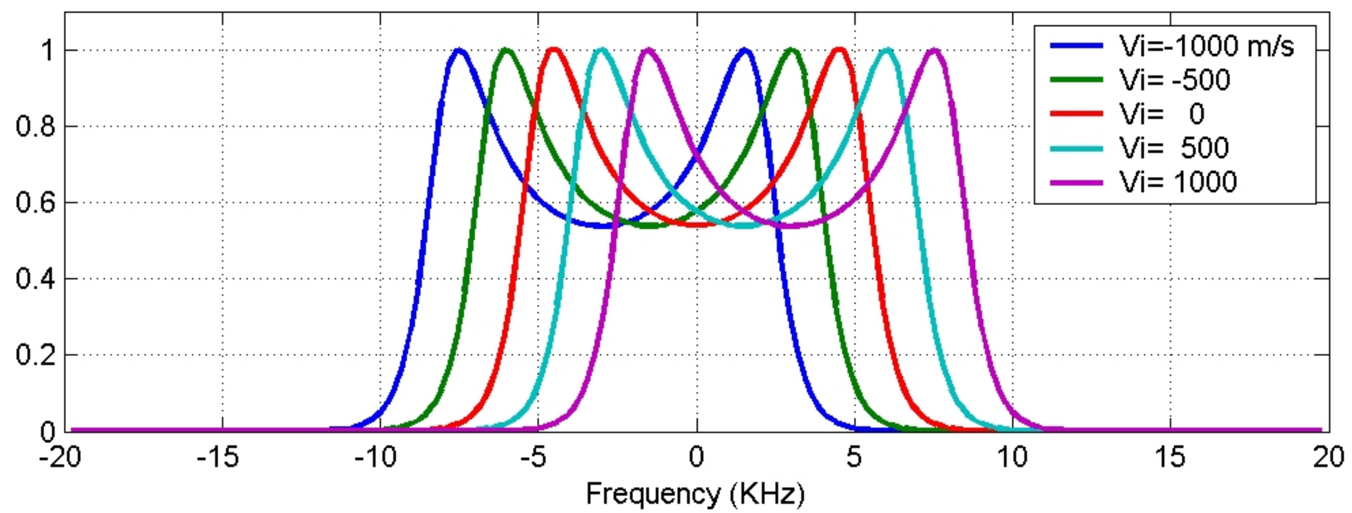
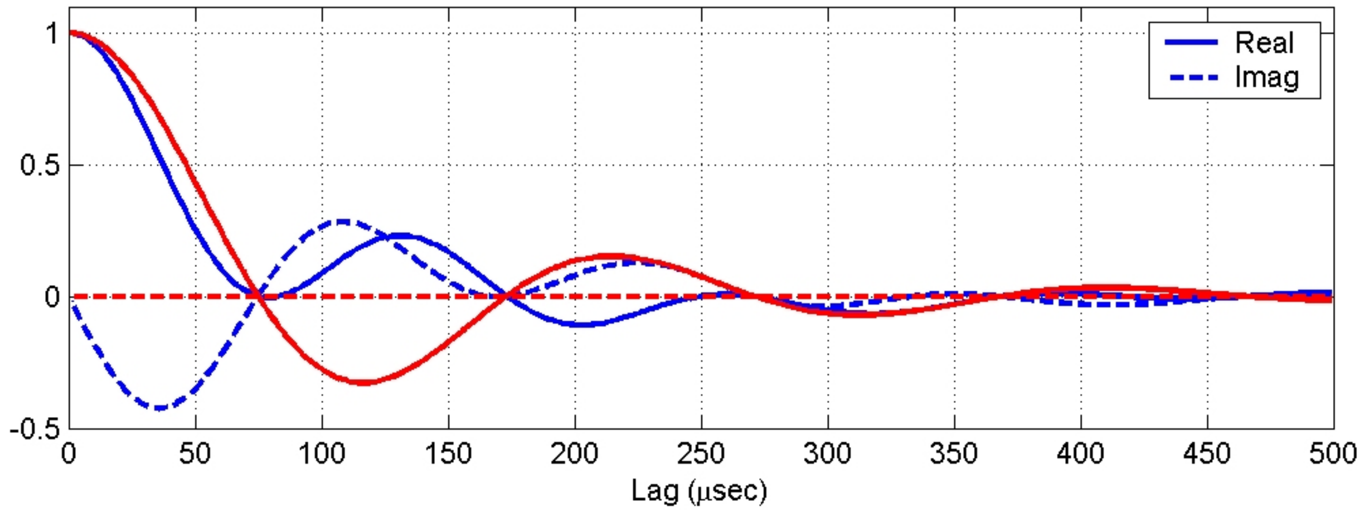
Ion Velocity





Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Te: 2000 K
Comp: 100% O^+
ν_{in}: 10^{-6} KHz

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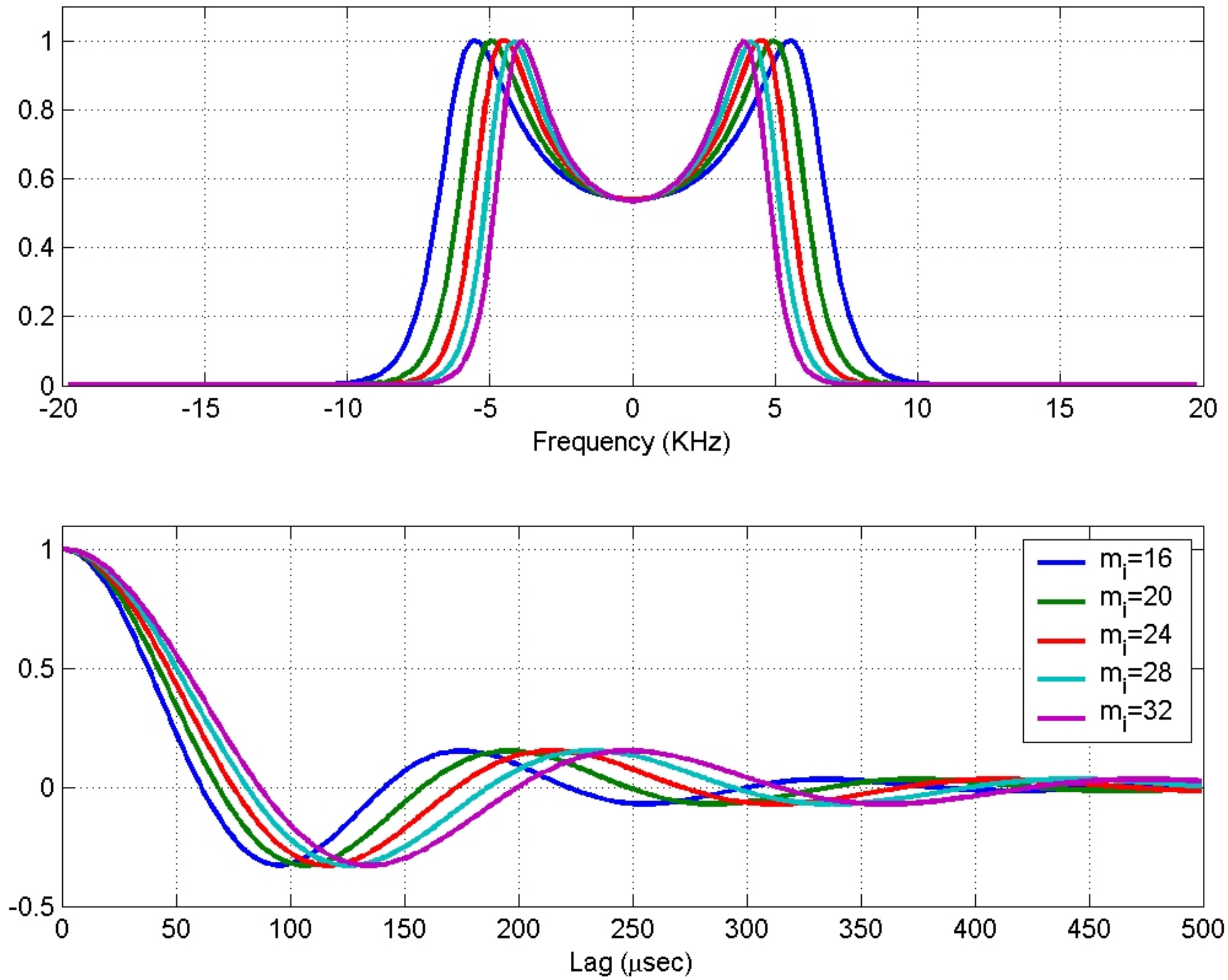
Ion Mass



Parameters

Freq: 449 MHz

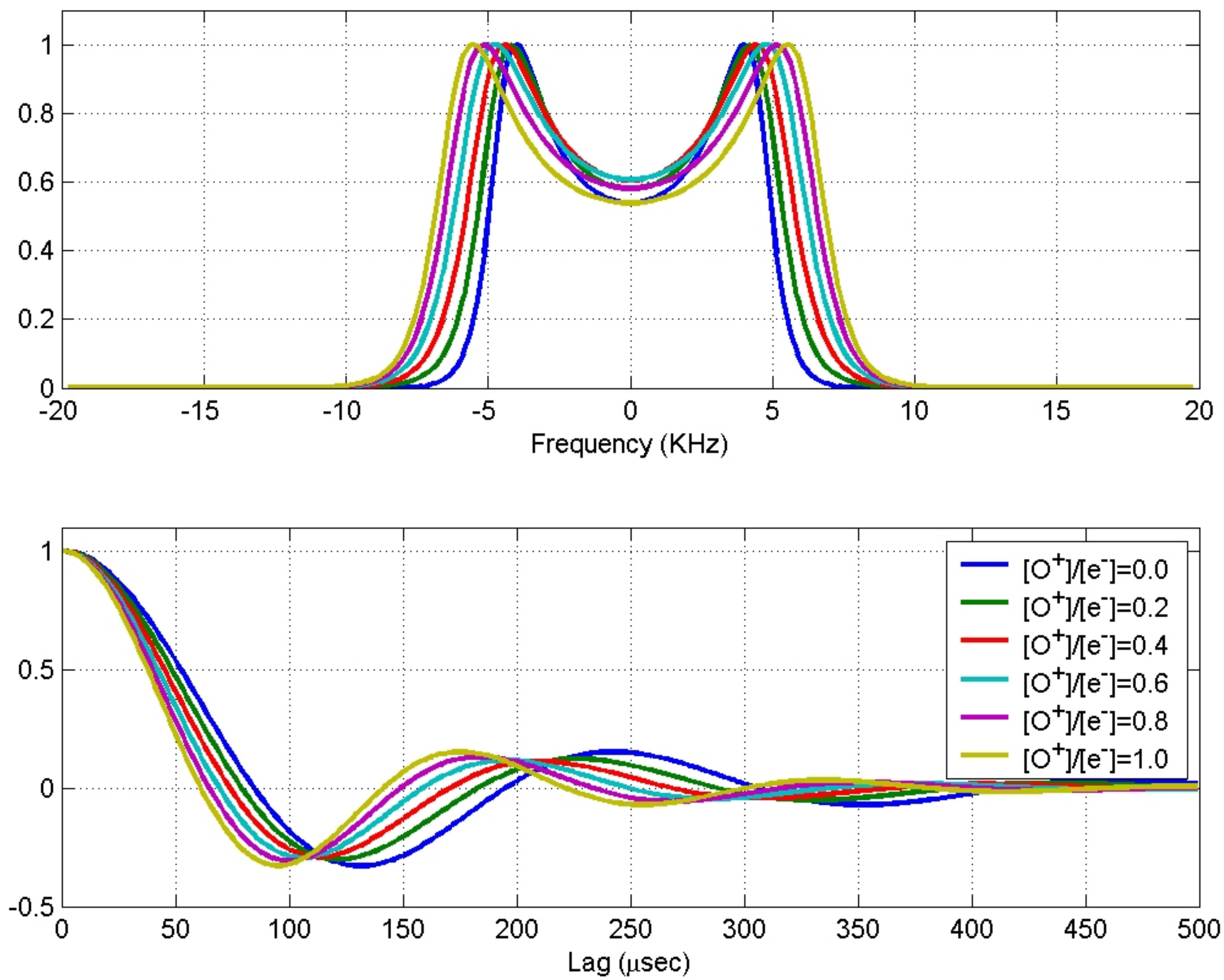
Ne: 10^{12} m^{-3}

Ti: 1500 K

Te: 3000 K

ν_{in}: 10^{-6} KHz

Ion Composition (O^+ vs. NO^+)



<u>Parameters</u>

Freq: 449 MHz

Ne: 10^{12} m^{-3}

Ti: 1500 K

Te: 3000 K

ν_{in}: 10^{-6} KHz

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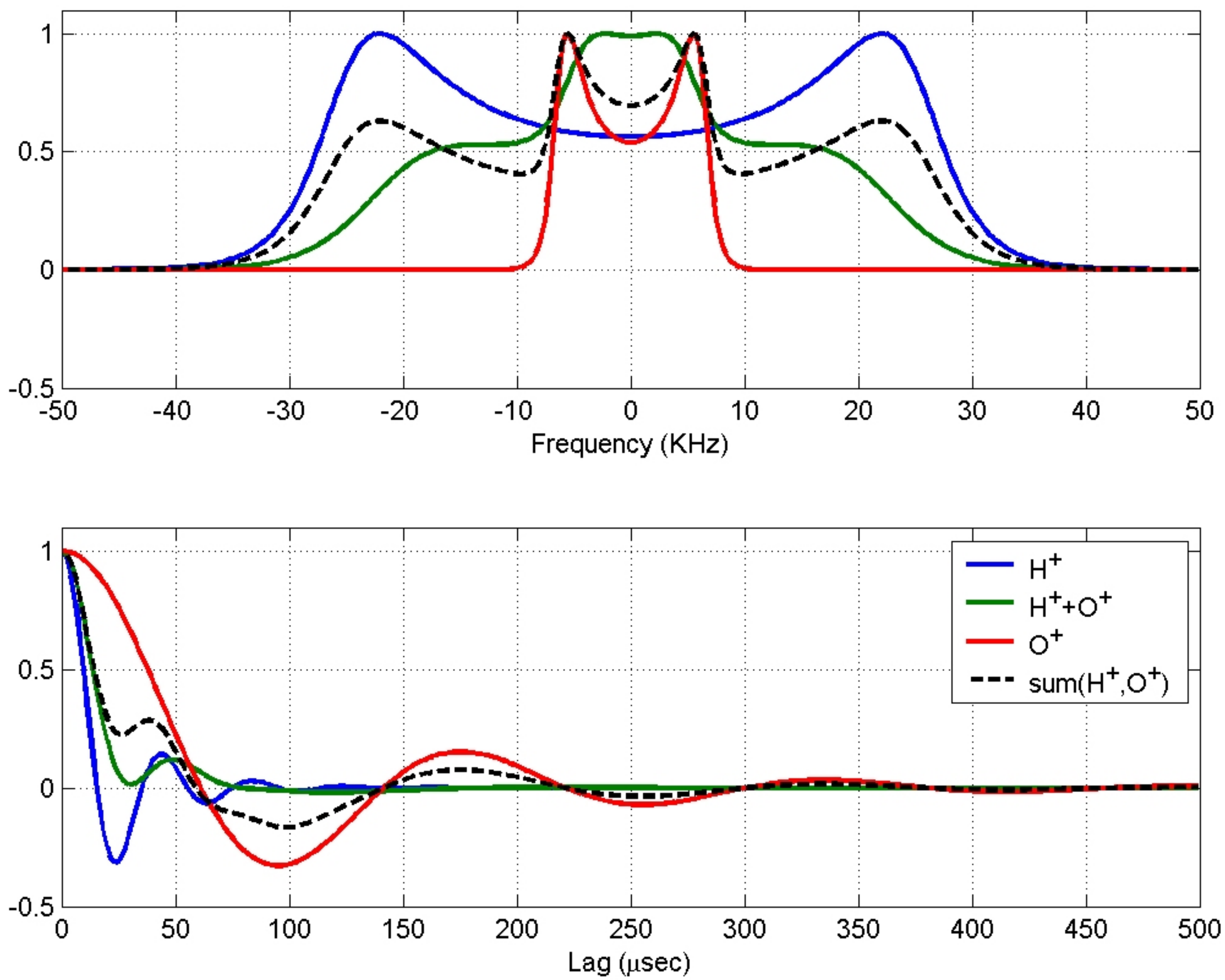
Ion Composition (O^+ vs. H^+)



Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1500 K
Te: 3000 K
ν_{in}: 10^{-6} KHz

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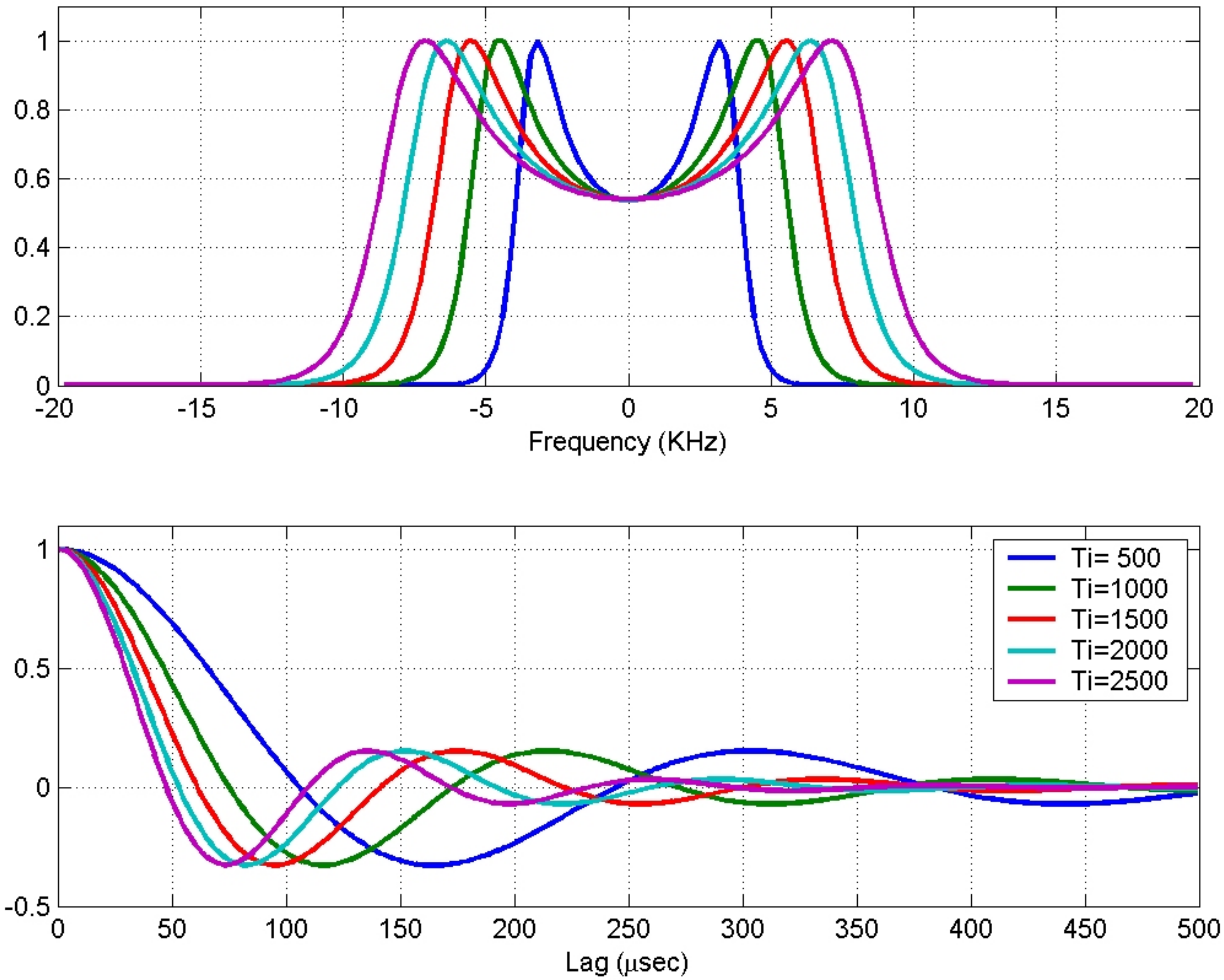
Ion Temperature



<u>Parameters</u>

Freq: 449 MHz

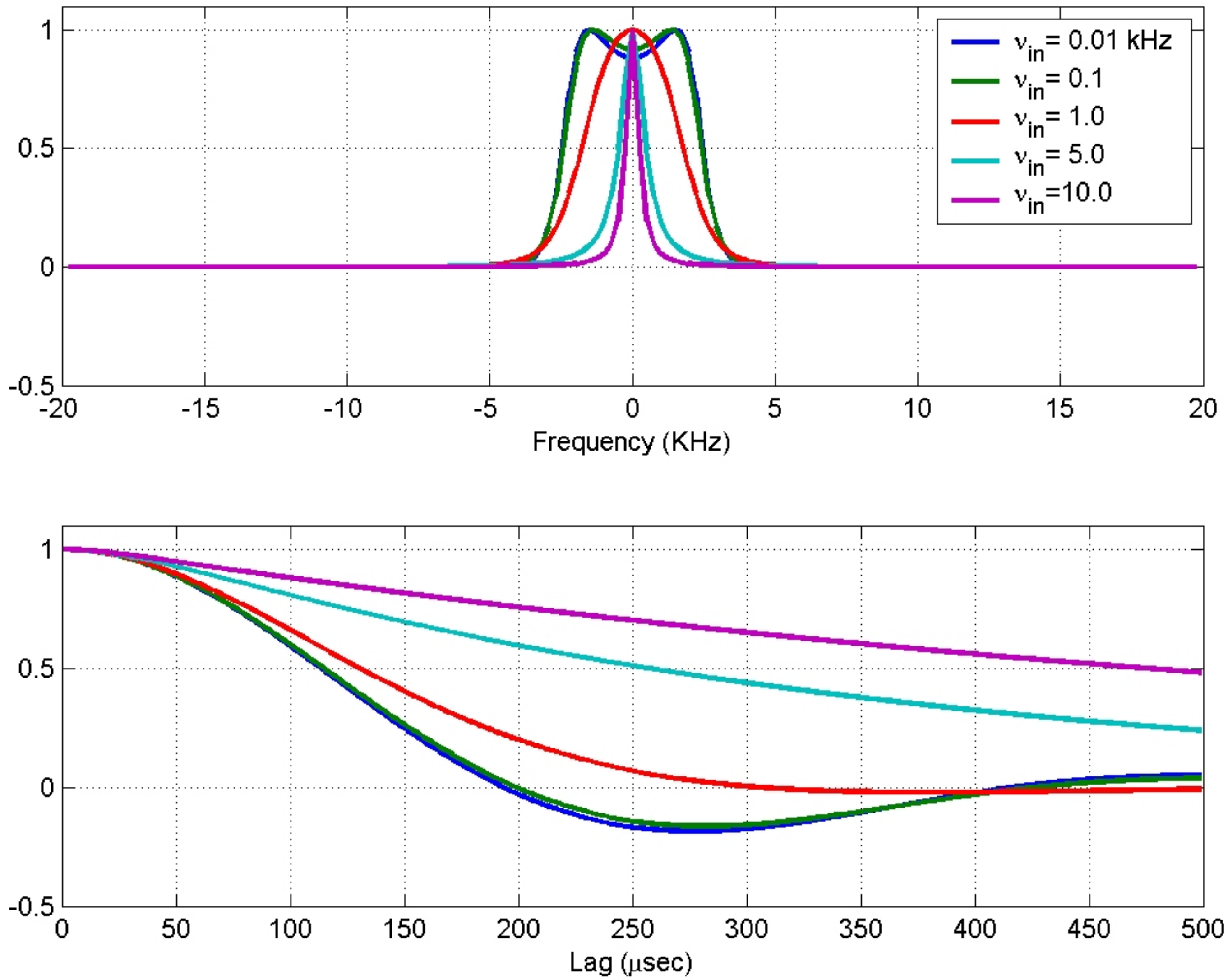
Ne: 10^{12} m^{-3}

Te: 2*Ti

Comp: 100% O^{+}

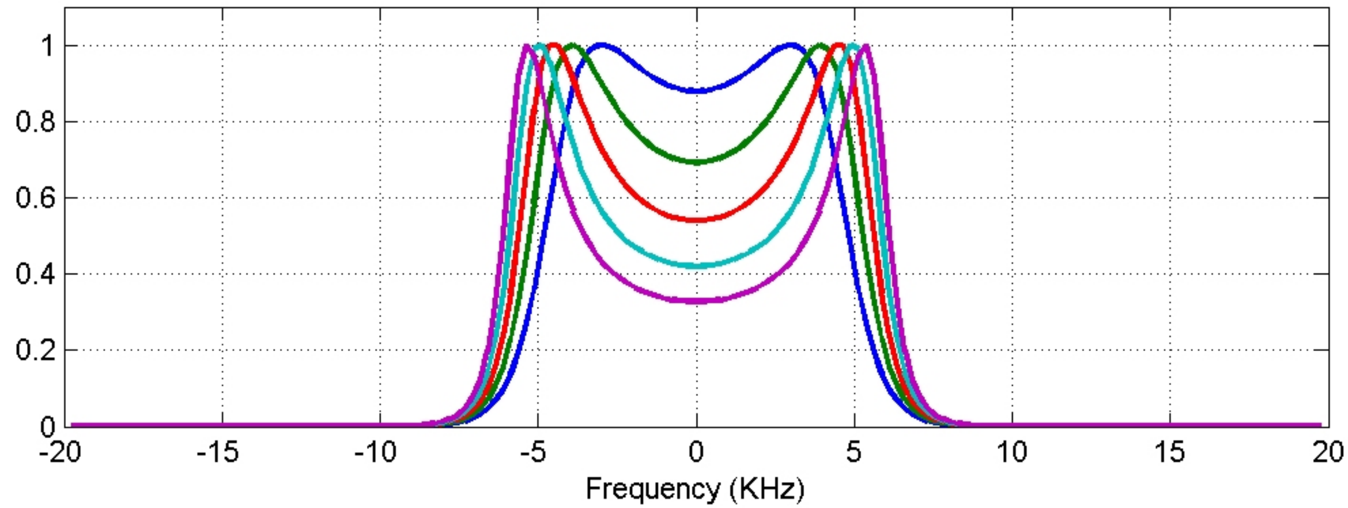
ν_{in}: 10^{-6} KHz

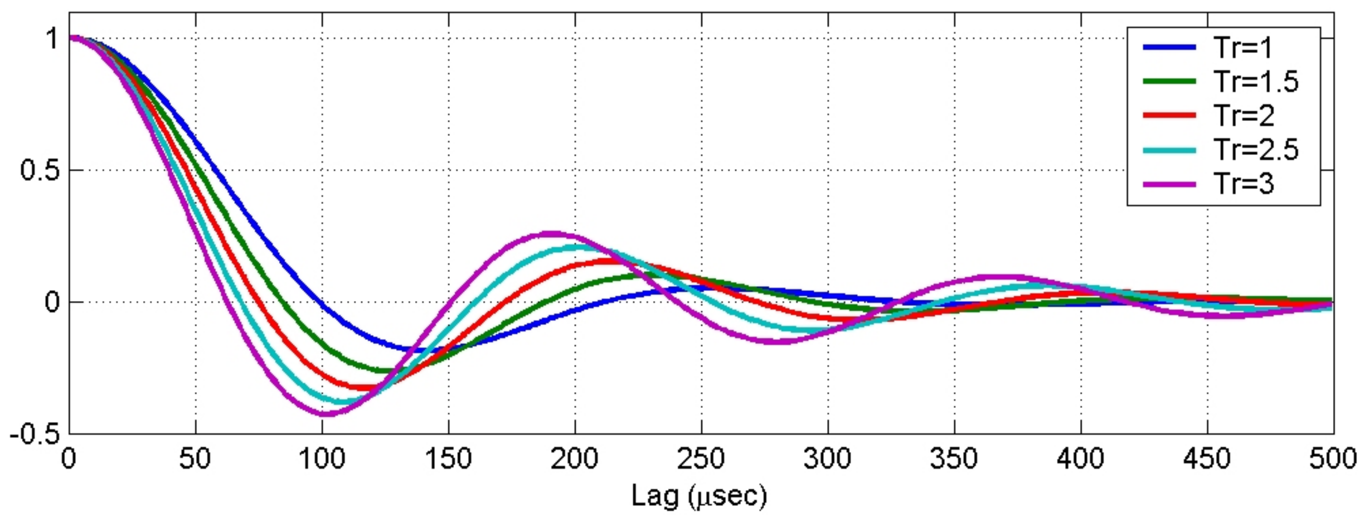
Ion-Neutral Collision Frequency



<u>Parameters</u>
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 500 K
Te: 500 K
Comp: 100% NO^+

Electron/Ion Temperature Ratio





Parameters
Freq: 449 MHz
Ne: 10^{12} m^{-3}
Ti: 1000 K
Comp: 100% O^{+}
ν_{in}: 10^{-6} KHz

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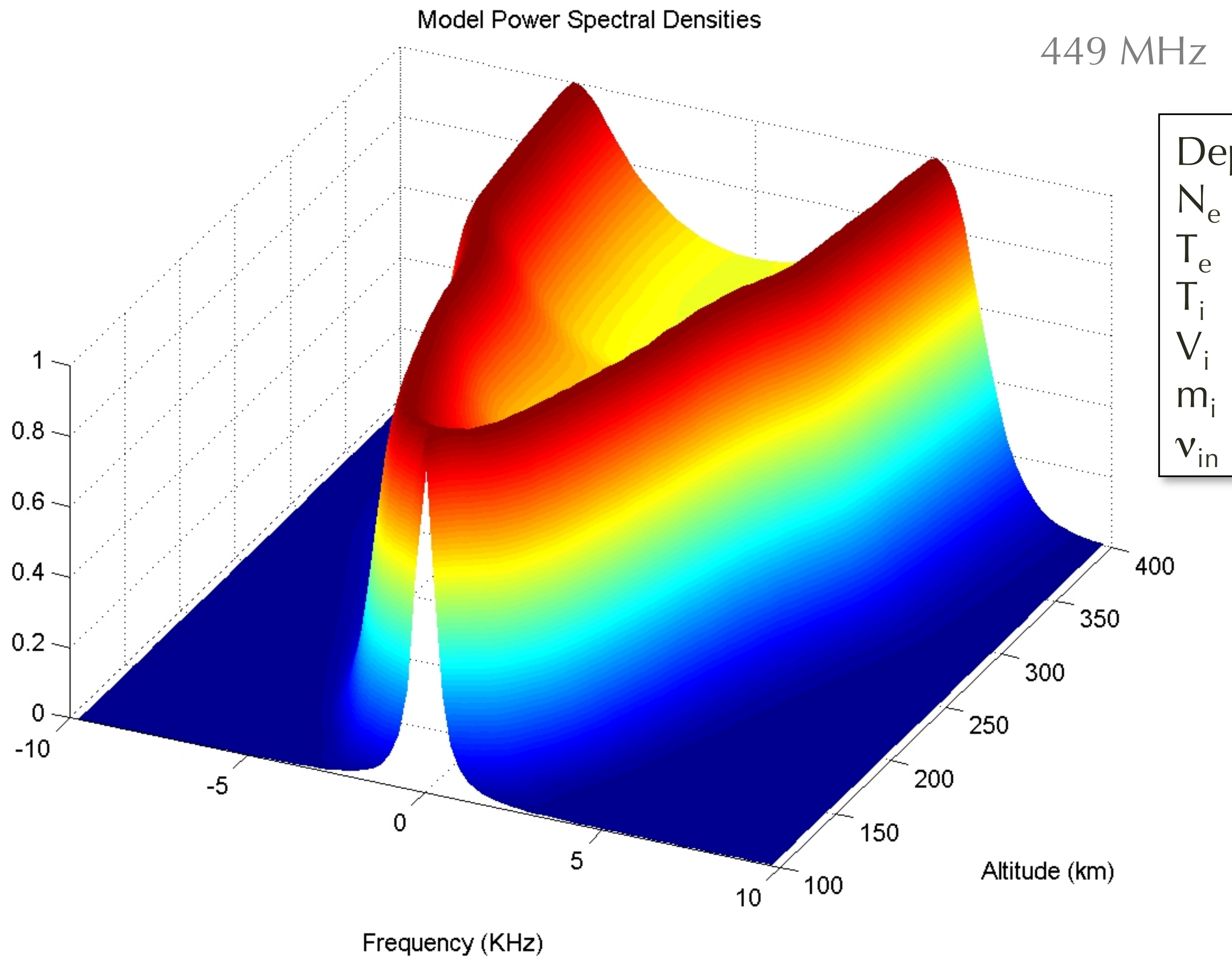
Incoherent Scatter PSD



Dependencies:
N_e
T_e
T_i
V_i
m_i
ν_{in}

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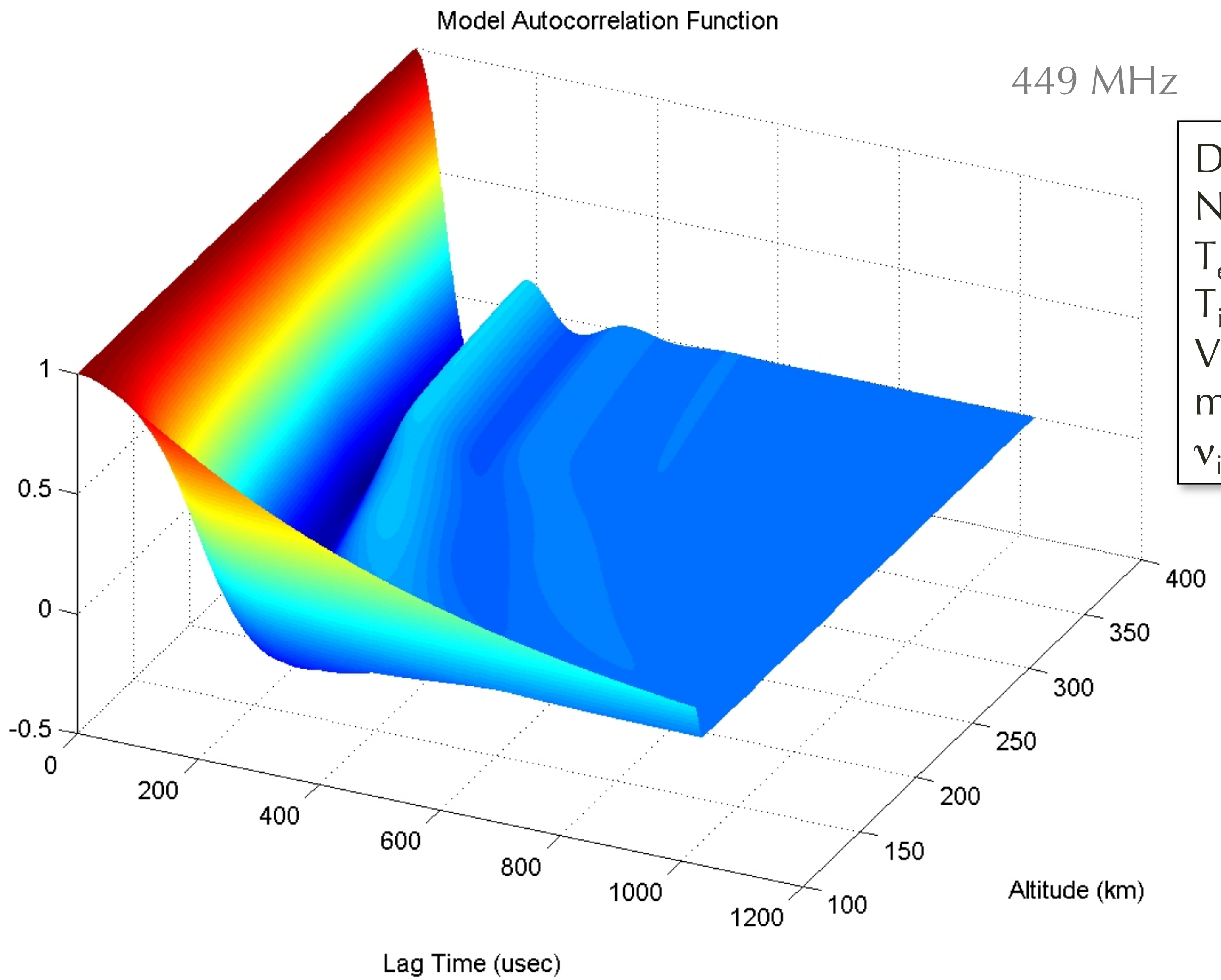
Incoherent Scatter ACFs



Dependencies:

N_e

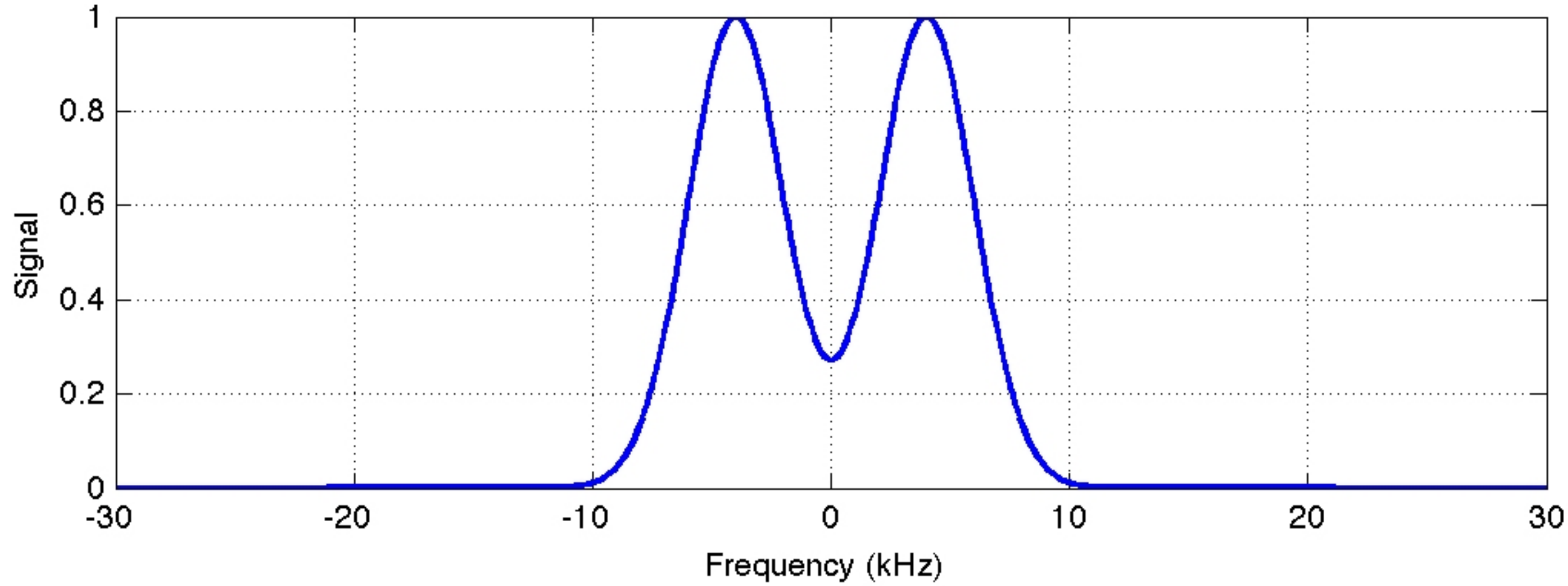
T_e

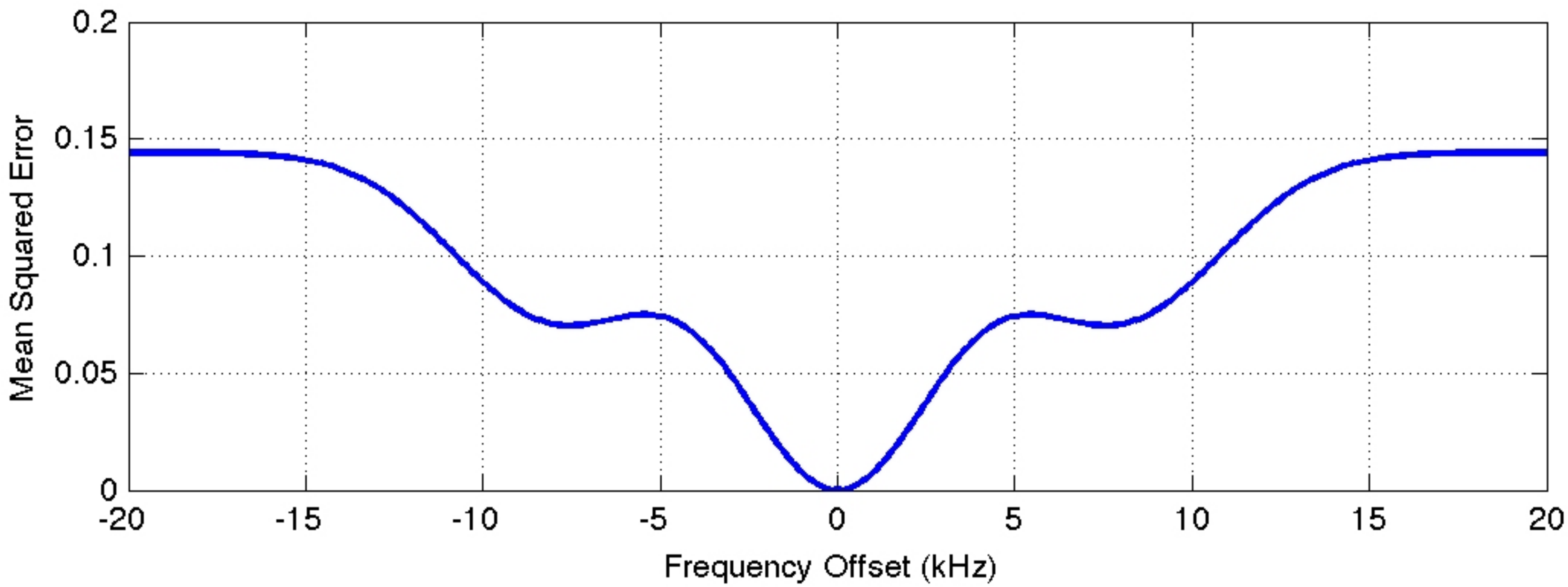
T_i

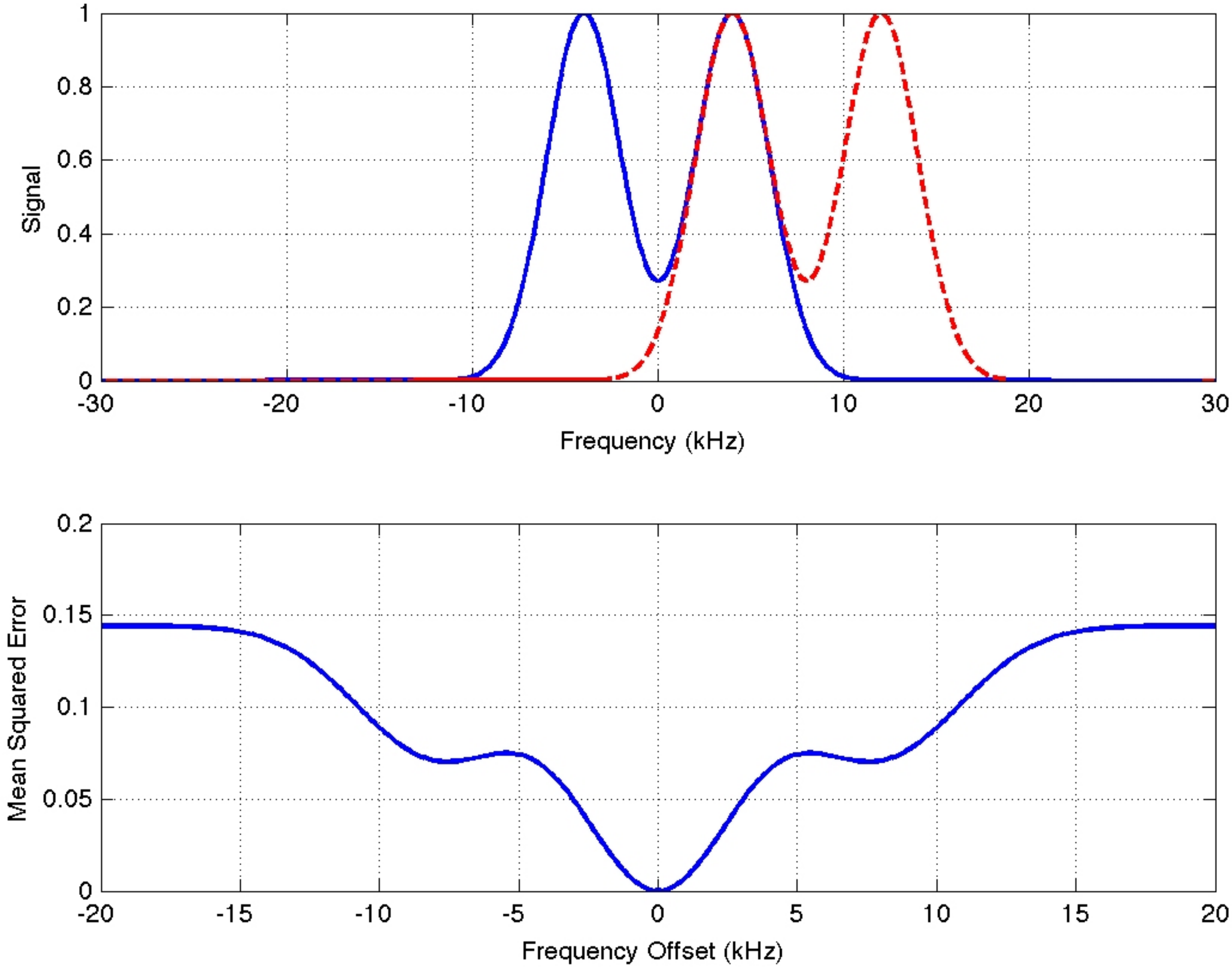
V_i

m_i

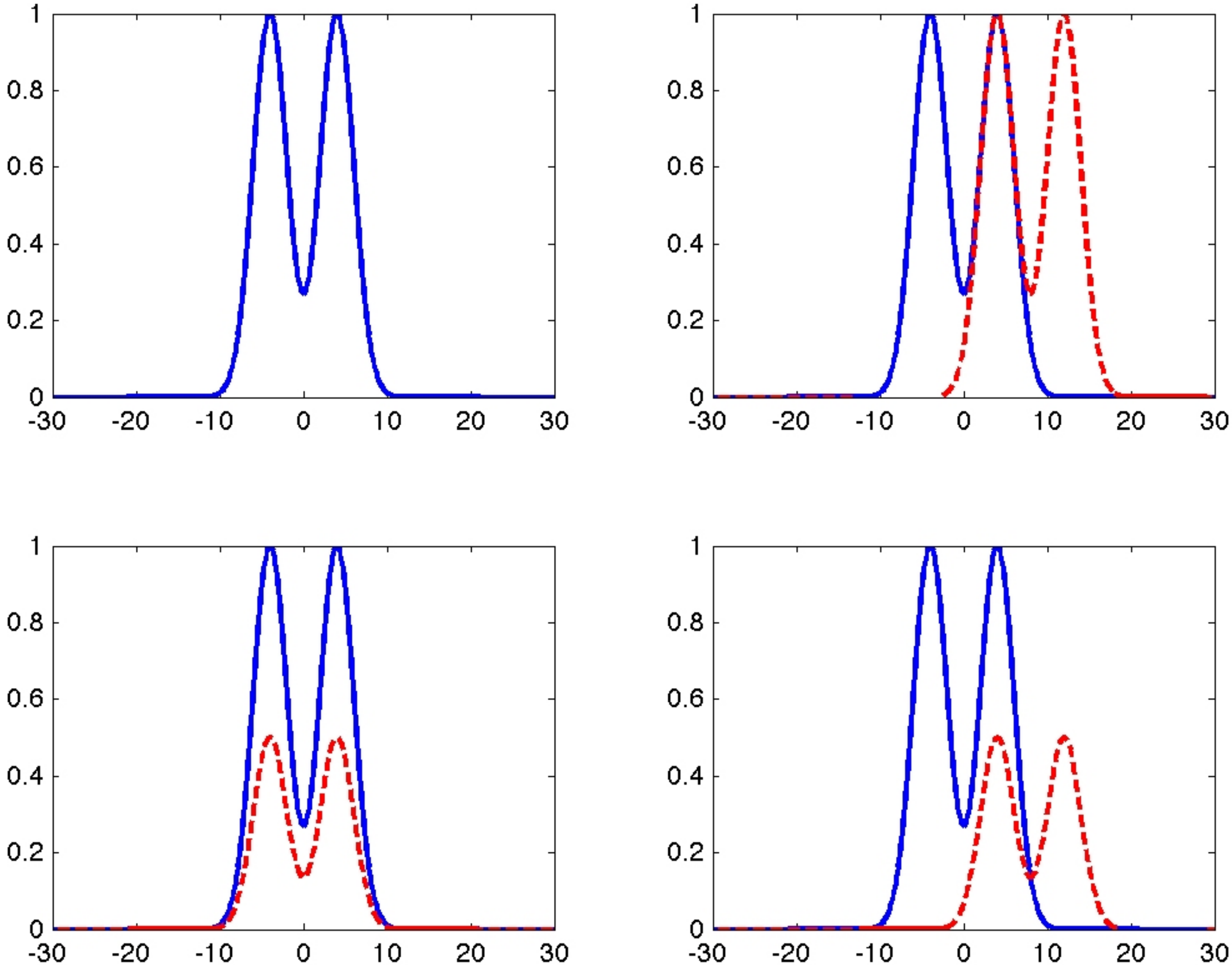
ν_{in}


Signal
Frequency (kHz)
1
0.8
0.6
0.4
0.2
0
-30
-20
-10
0
10
20
30

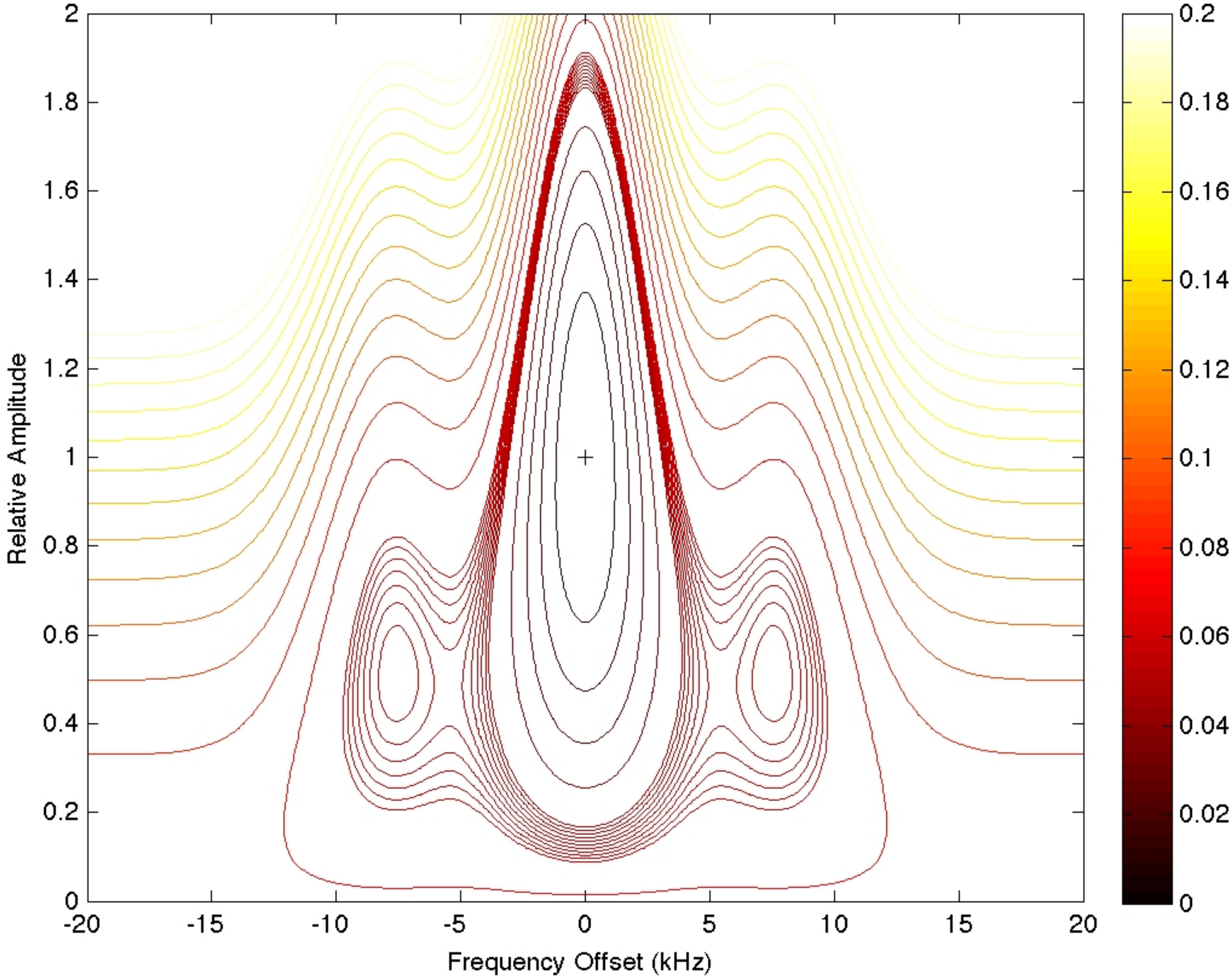

Mean Squared Error
Frequency Offset (kHz)
0.2
0.15
0.1
0.05
0
-20
-15
-10
-5
0
5
10
15
20



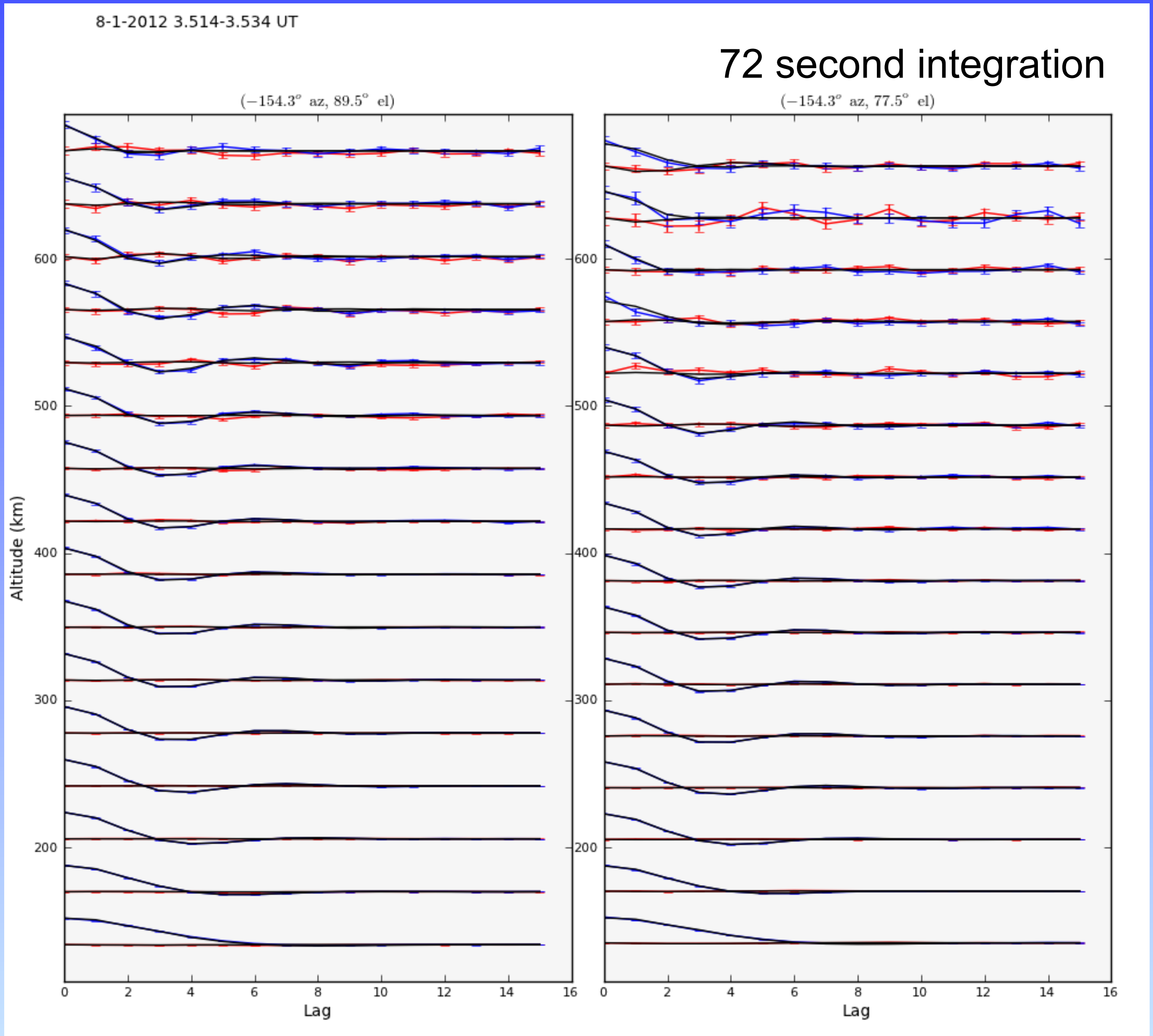
Signal
1
0.8
0.6
0.4
0.2
0
-30
-20
-10
0
10
20
30
Frequency (kHz)
Mean Squared Error
0.2
0.15
0.1
0.05
0
-20
-15
-10
-5
0
5
10
15
20
Frequency Offset (kHz)

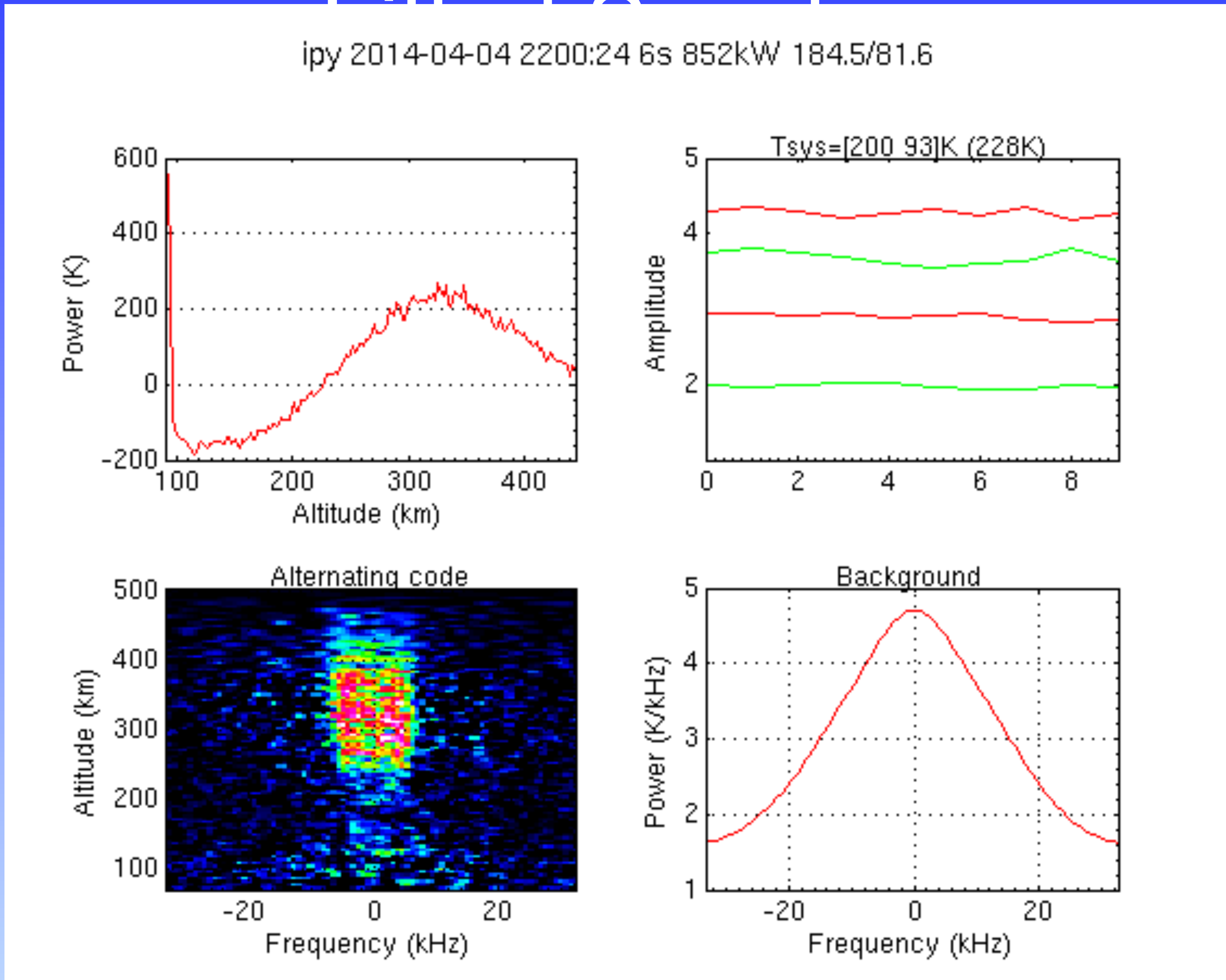


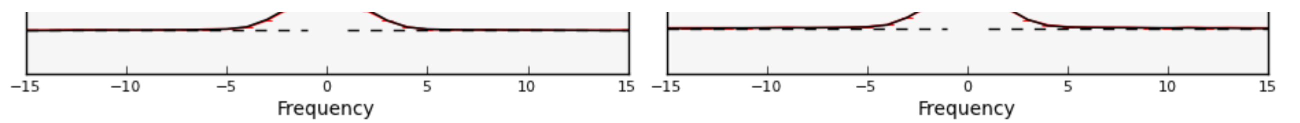
1
0.8
0.6
0.4
0.2
0
-30
-20
-10
0
10
20
30


Relative Amplitude
Frequency Offset (kHz)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-20
-15
-10
-5
0
5
10
15
20
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0

Fitted ACFs from PFISR




ipy 2014-04-04 2200:24 6s 852kW 184.5/81.6
Power (K)
Altitude (km)
Tsys=[200 93]K (228K)
Amplitude
Alternating code
Altitude (km)
Frequency (kHz)
Background
Power (K/kHz)
Frequency (kHz)


Frequency
Frequency

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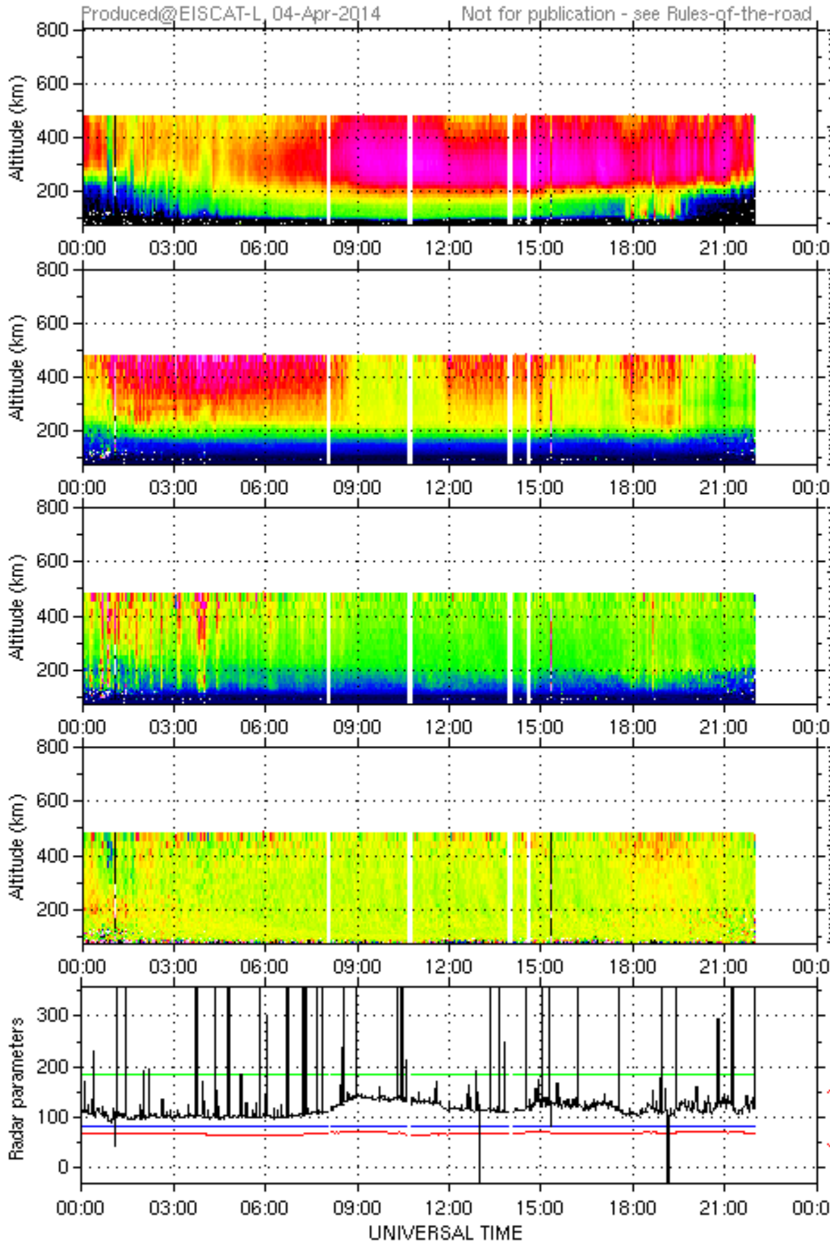
EISCAT SVALBARD RADAR

CP, 42m, ipy, 4 April 2014



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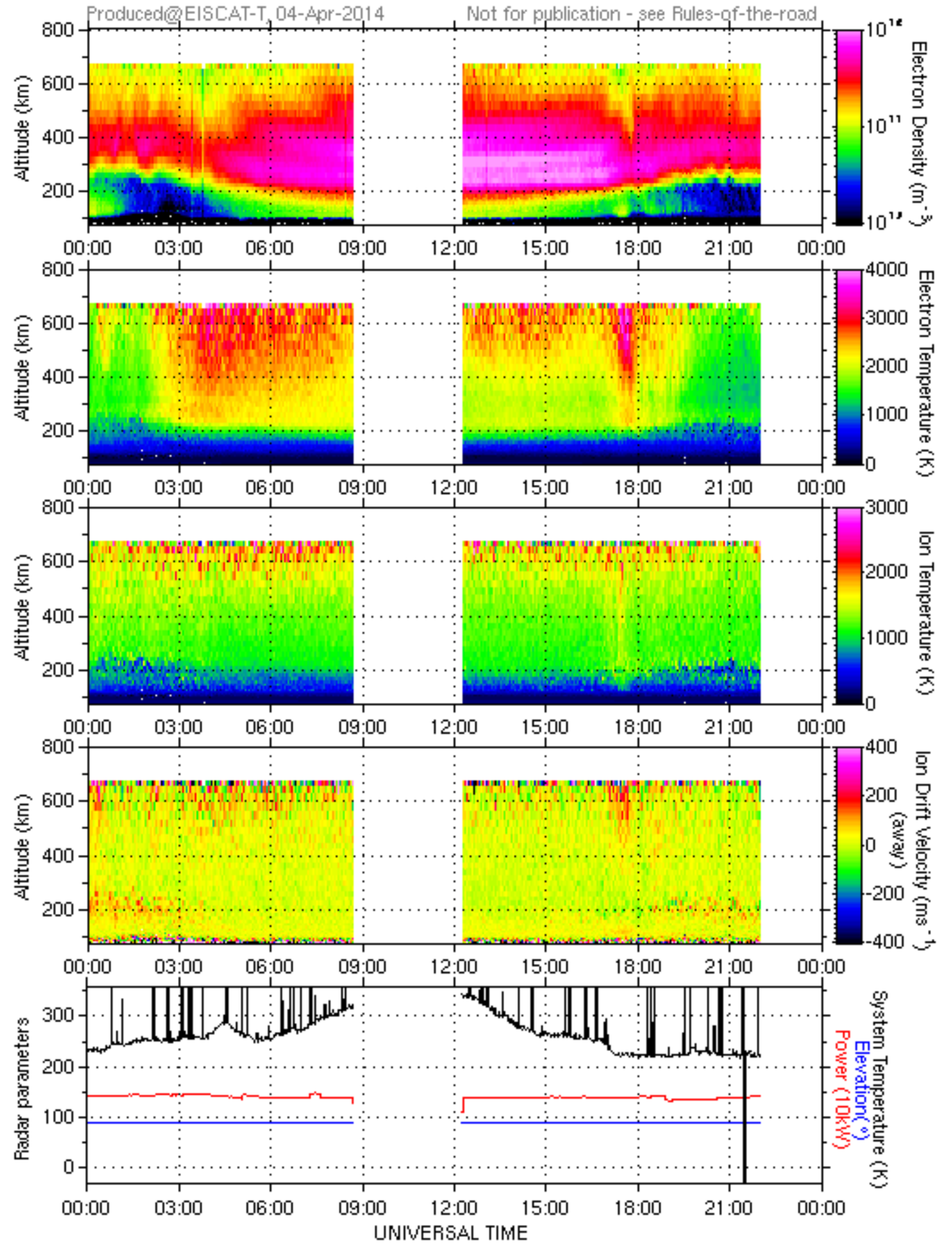
EISCAT VHF RADAR

CP, vhf, beata, 4 April 2014



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Thank You

...for putting the i in MST!