Radar Interferometric Imaging

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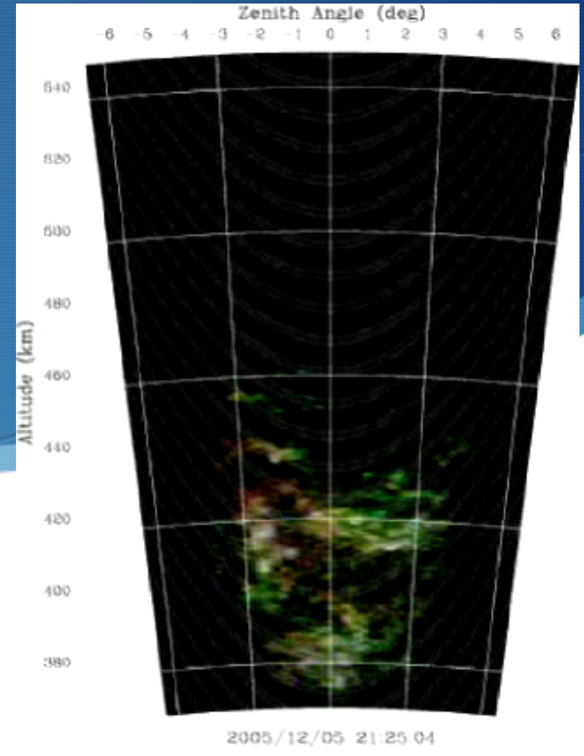
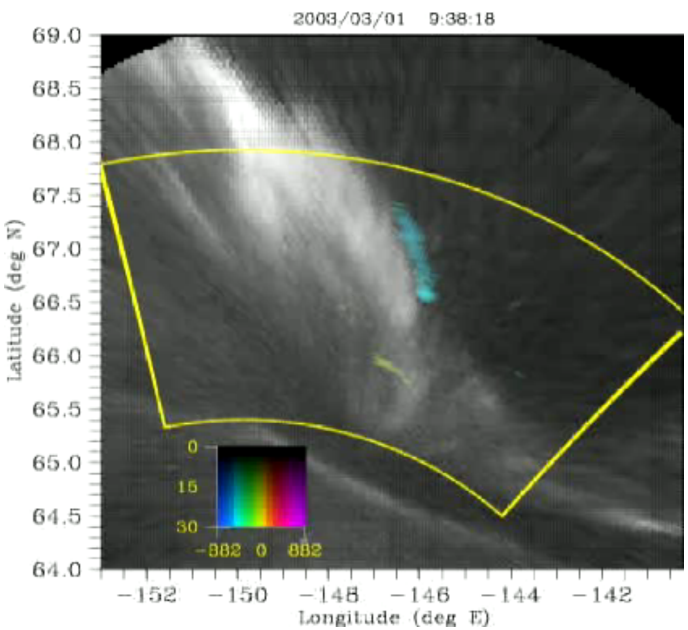
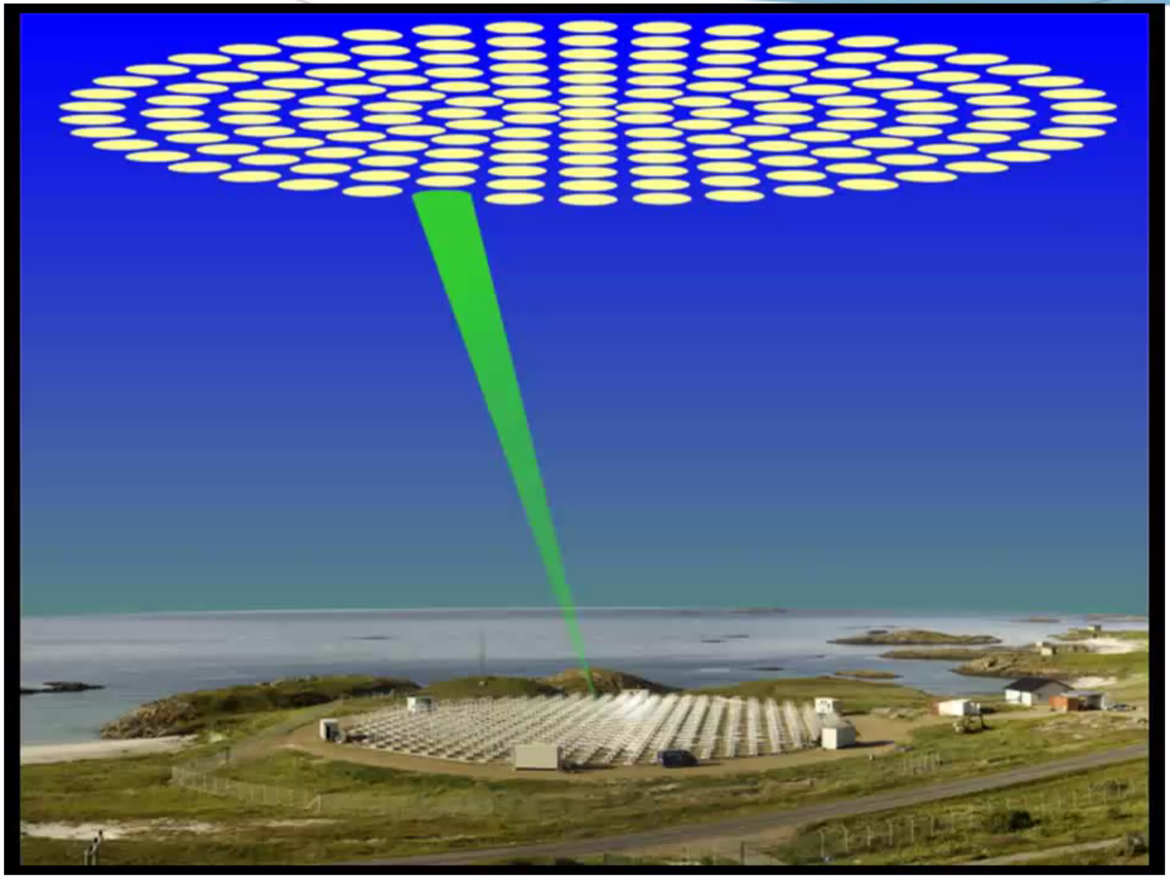
Outline

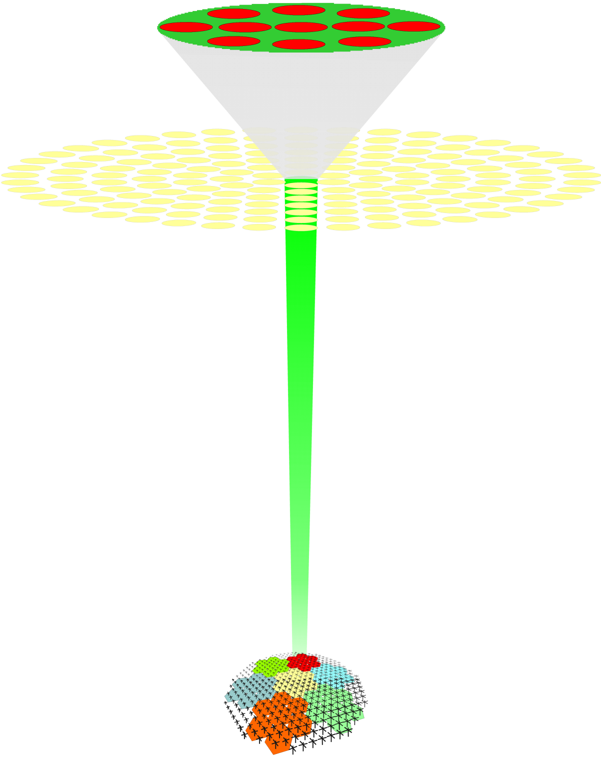
- Motivation
  - Equatorial Ionospheric Irregularities
  - Wide beam “targets”
- Radio Interferometry basics
- Aperture synthesis for Ionospheric applications
  - Methods
  - Phase calibration
  - Examples (EEJ, ESF, 150 km, QP, Aurora)
- “JP” Imaging modes
- Latest developments





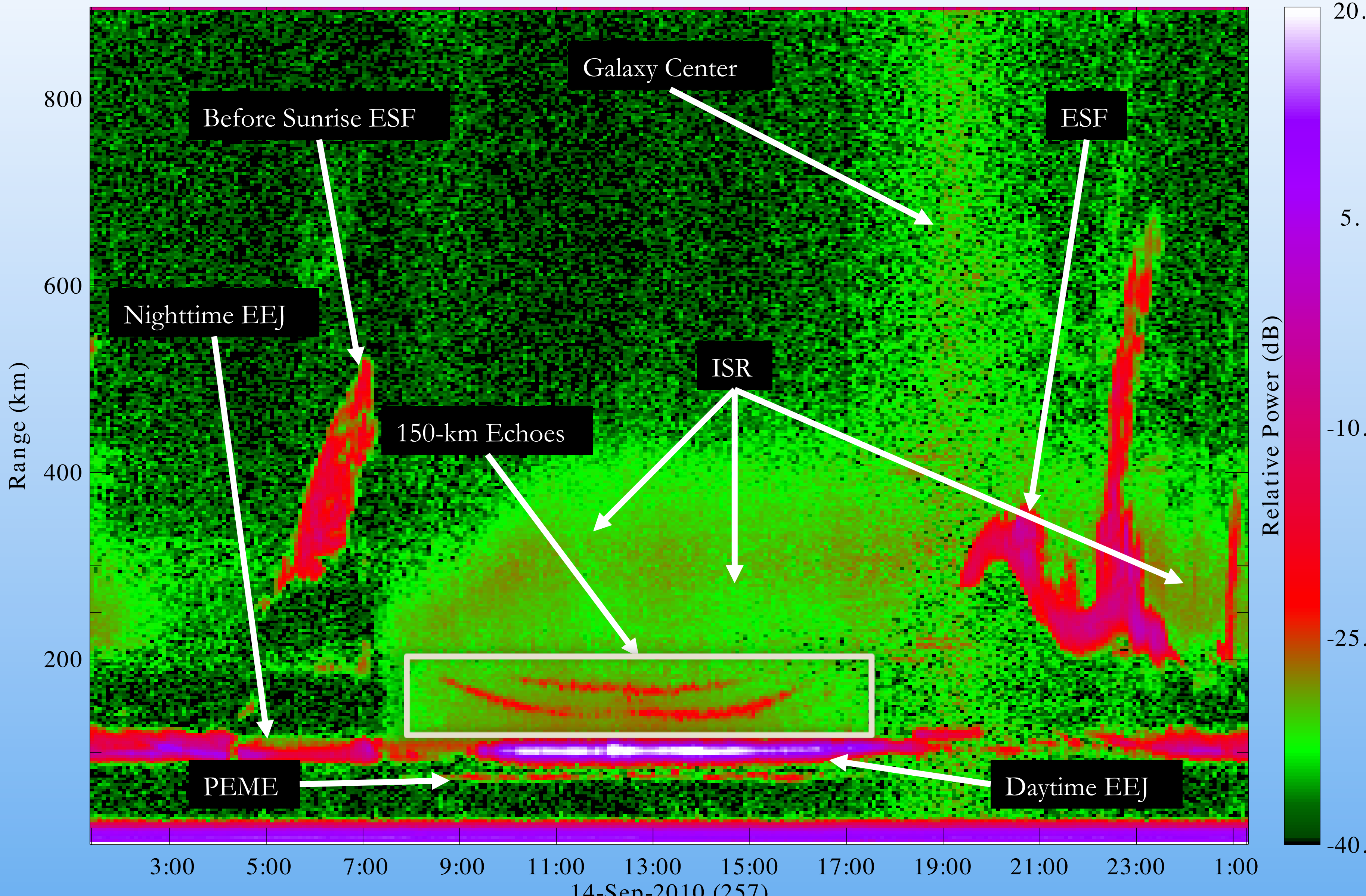
Digital beam steering vs. In-beam Imaging





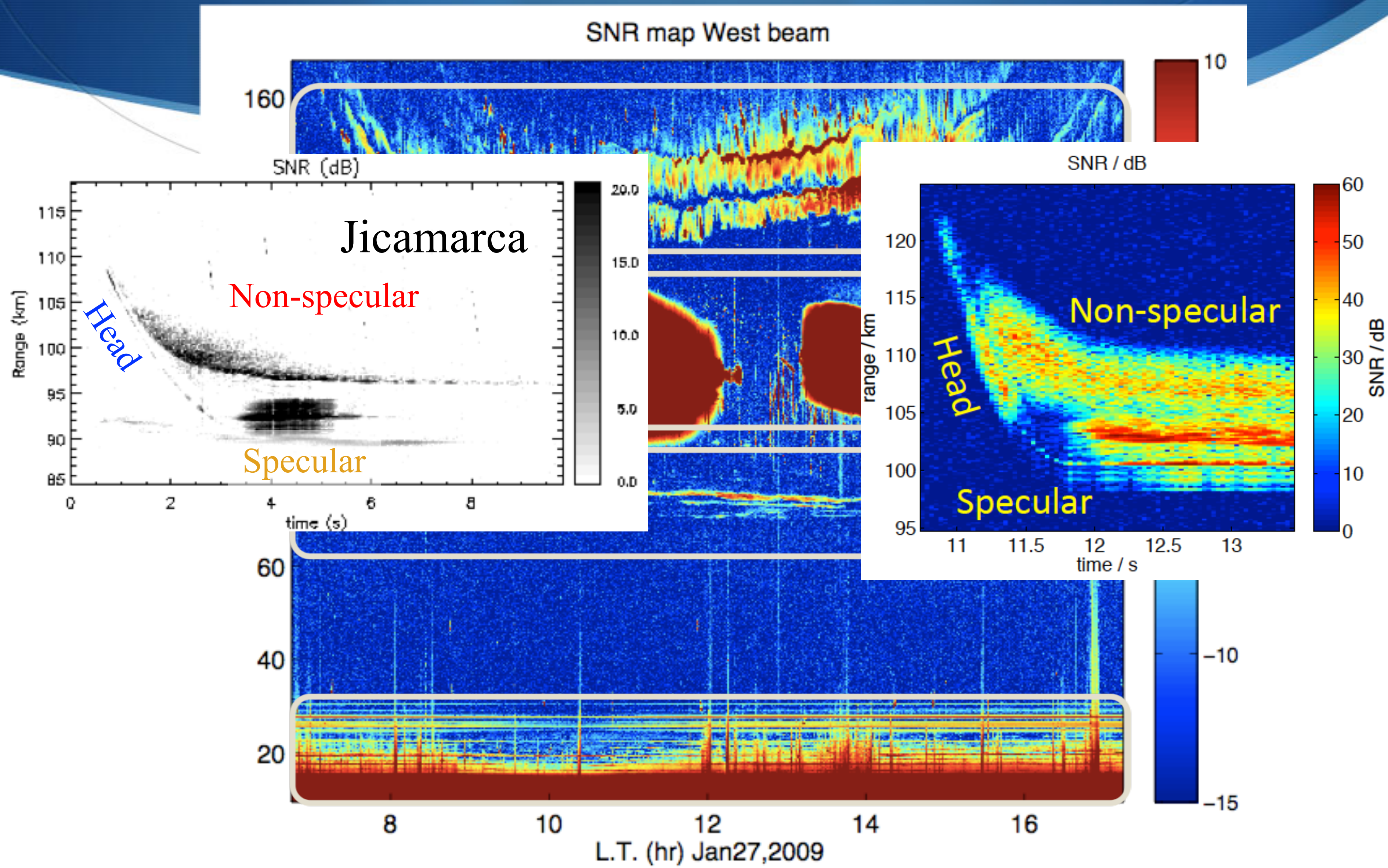
$$P_r \propto \frac{P_t}{4\pi r^2} \int d\theta_x \, d\theta_y G_t(\theta,\phi) G_r(\theta_x,\theta_y) \sigma(r,\theta_x,\theta_y,\omega)$$

Incoherent and Coherent Echoes over Jicamarca

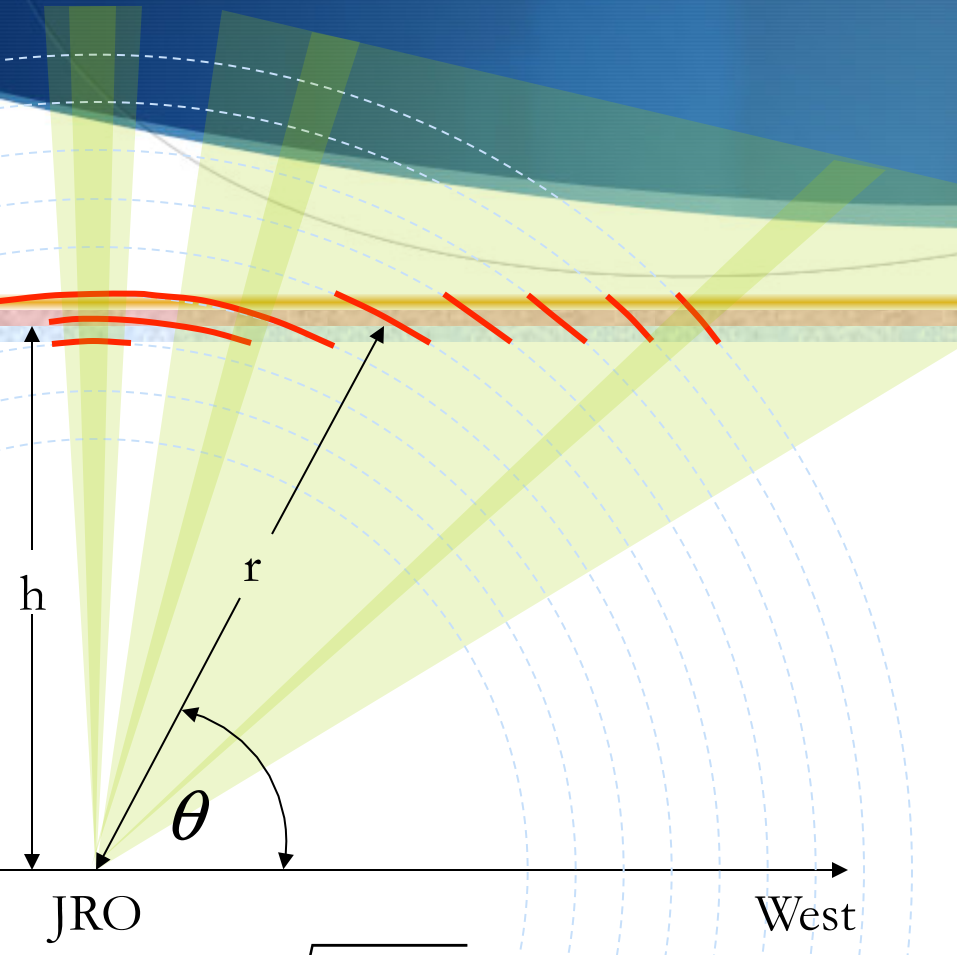


[Courtesy Juha Vierinen]

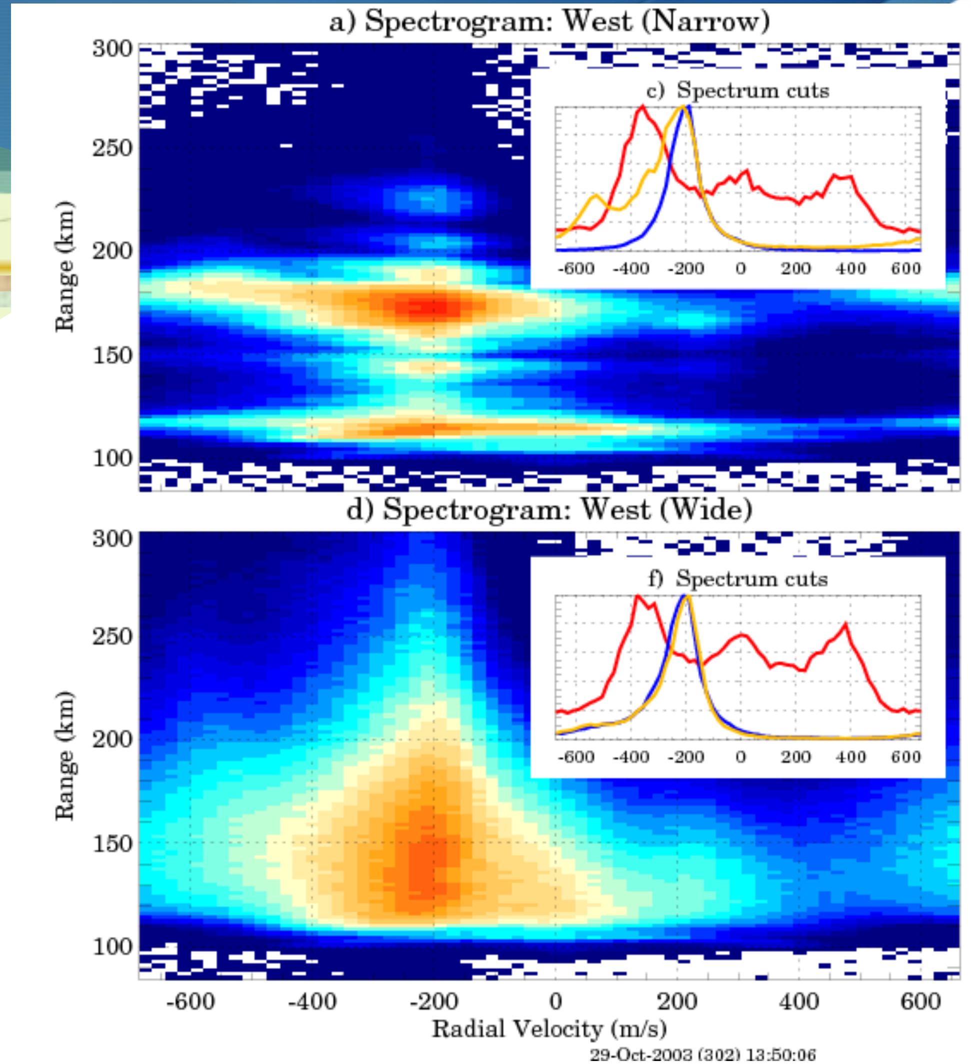
Equatorial Irregularities below 200 km



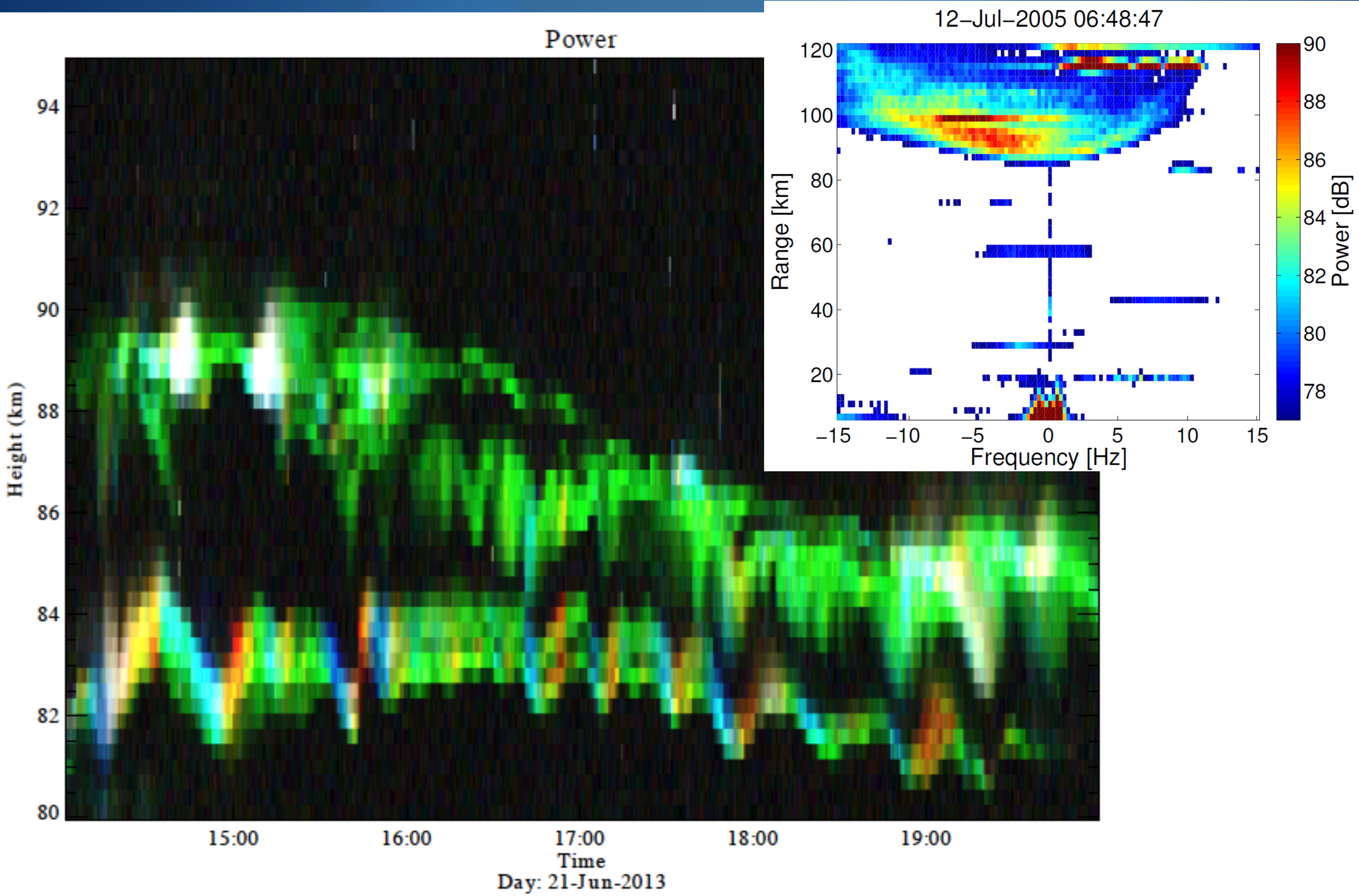
EEJ Wide beam Observations: Geometry and Spectra



$$\cos\theta = \frac{\sqrt{r^2 - h^2}}{r}$$



PMSE Narrow and Wide beam observations



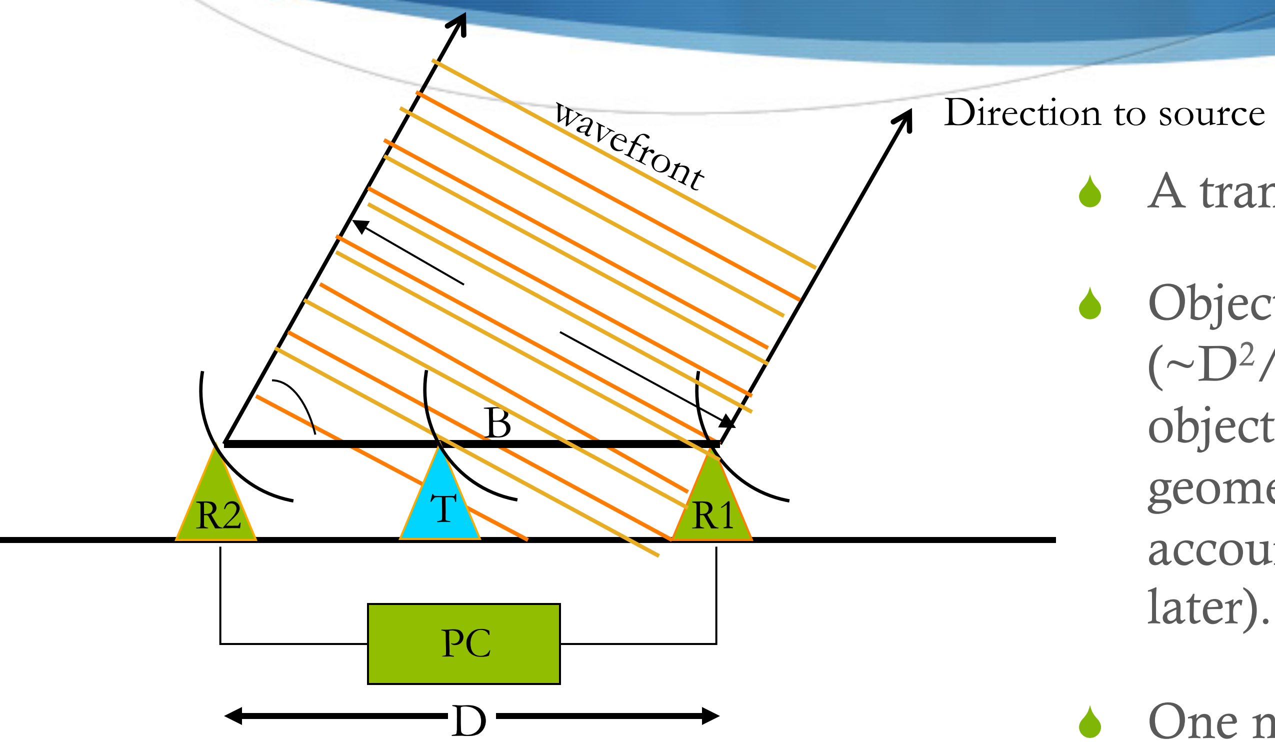
What can we get from single beam radar measurements?

- If the scattering is volume-filling, i.e., the whole illuminated volume presents the same statistical characteristics (space and time), then the information is contained in the spectrum. For example, in incoherent scatter spectrum, the spectra shape determines the temperature of ions and electrons, and also composition.

- If the scattering is not volume filling (the case for most coherent echoes), then the single beam information provides a weighted average information inside the illuminated volume. Is it possible to get spatial information?

- Note: Coherent echoes are 20-60 dB stronger than typical ISR echoes.

Introduction: Two-element Radar Interferometer



- A transmitted signal is needed.
- Object is located in the far-field ($\sim D^2/\lambda$). For Near field objects, tx as well as rx geometry need to be taken into account (see specific details later).
- One measures the cross-correlation (Amplitude and phase) between spatially separated receiving antennas.

RI: Single Scatterer

$$V(x,y,t) = Ae^{j(\omega t + k_0\theta_x x + k_0\theta_y y)}$$

- The output of coherent detector connected to an antenna at (*x,y, z=0*) is some

$$\begin{aligned} R(\xi,\eta,\tau) &\equiv V^*(x,y,t)V(x+\xi,y+\eta,t+\tau) \\ &= |A|^2 e^{j(\omega\tau + k_0\theta_x\xi + k_0\theta_y\eta)} \end{aligned}$$

that describes travelling co-sinusoids on the *xy*-plane with a frequency ω and wavenumbers $k_0\theta_x$ and $k_0\theta_y$ related to target motion and position.

- With Radar Interferometry, these target parameters can be extracted from the phase angle of conjugate products pairs such as

Using only three spaced antennas at $(x,y) = (0,0), (\xi,0)$, and $(0,\eta)$,

$$\theta_x = \frac{\measuredangle R(\xi,0,0)}{k_0\xi}, \quad \theta_y = \frac{\measuredangle R(0,\eta,0)}{k_0\eta}, \quad \omega = \frac{\measuredangle R(0,0,\tau)}{\tau} = -2k_0 v_r$$

RI Theory: Multiple or extended scatterers

$$V(x,y,t) = \int d\theta_x d\theta_y df\, A(\theta_x,\theta_y,f)e^{j(2\pi ft + k_0\theta_x x + k_0\theta_y y)}$$

- The incident voltage for a spectrum of plane waves is given by

Where $A(\theta_x,\theta_y,f)$ represents plane waves with Doppler frequencies $f = \omega/2\pi$ and directions of arrivals θ_x,θ_y

$$\mathrm{PDF}[\theta_i] \propto e^{-\left(\theta_i - \bar{\theta}\right)^2/(2\sigma_\theta^2)}$$

$$S_{EW}(\omega) = \frac{\left\langle V_E(\omega)V_W^*(\omega)\right\rangle}{\sqrt{\left\langle\left|V_E(\omega)\right|^2\right\rangle\left\langle\left|V_W(\omega)^2\right|\right\rangle}}$$

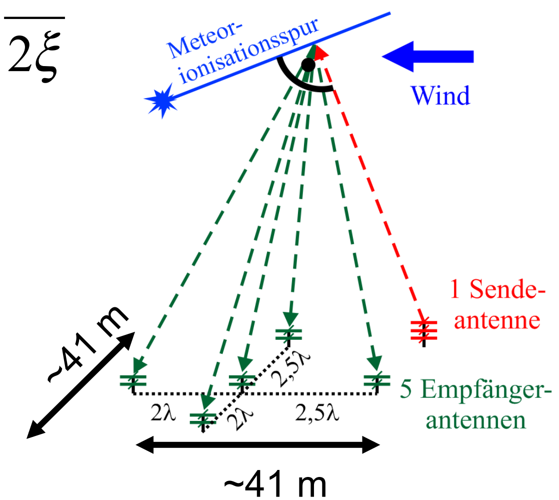
$$S_{EW}(\omega) = e^{jk_0\xi\bar{\theta}}e^{-\frac{1}{2}k_0^2\xi^2\sigma_\theta^2}$$

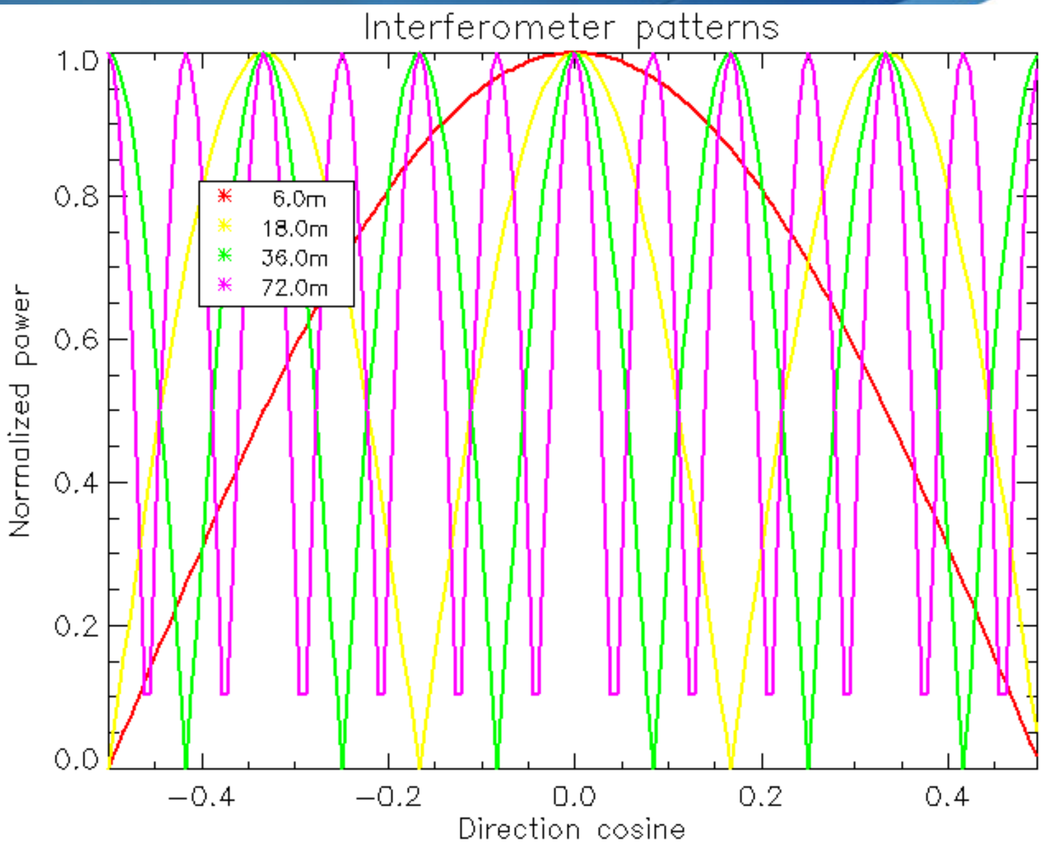
Processing and Pract.Considerations: Interferometer Pattern

$$\sin\theta = \frac{\measuredangle \left\langle R_{EW}(\xi) \right\rangle_t}{k_0 \xi}$$

since the maximum "unambiguous phase is π, then the maximum "unambiguous zenith angle" for a two-antenna interferometer is

$$\theta_{\max} = \sin^{-1}\frac{\lambda}{2\xi}$$





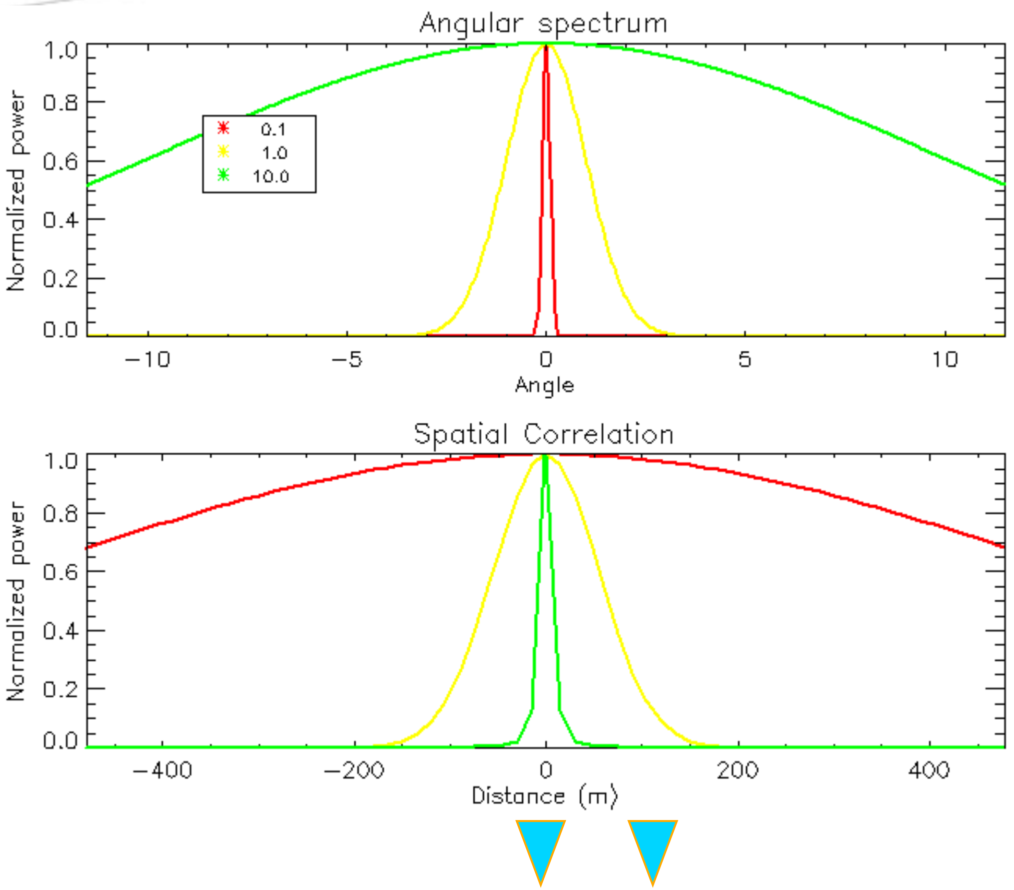
Specular meteor radars use the so-called Jones configuration

Processing and Pract.Considerations: Angle-Space, Fourier Pairs

Angle spectrum and Spatial correlation forms Fourier Pairs $\theta \Leftrightarrow \xi$ ($t \Leftrightarrow \omega$ in time-frequency analysis). Then, we can infer a number of pratical considerations for a our simple interferometer, e.g.,

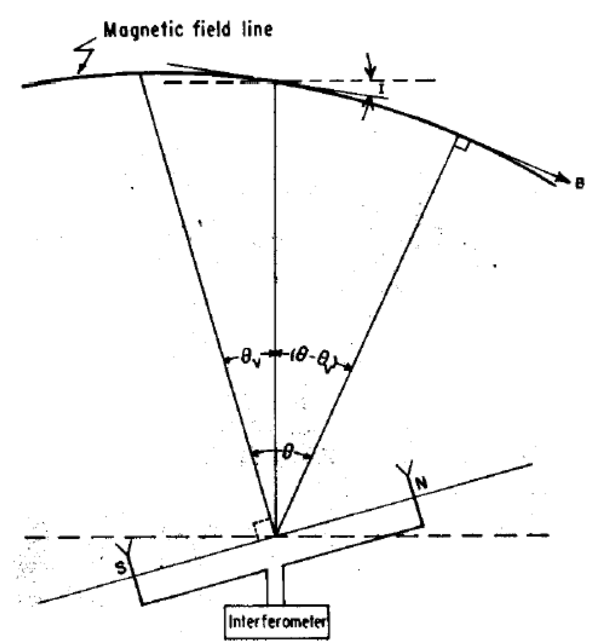
$$\theta_{\text{"Nyquist"}} \propto \frac{\lambda}{2\xi_{\min}} \quad \left(f_{\text{Nyquist}} = \frac{1}{2T_{sampling}}\right)$$

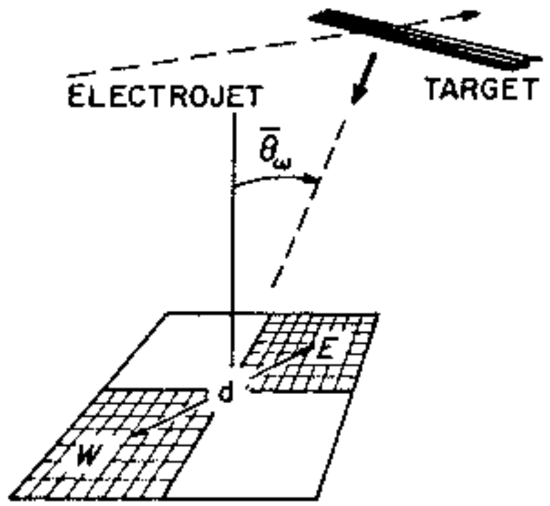
$$\theta_{\min} \propto \frac{\lambda}{\xi_{\max}} \quad \left(f_{\min} = \frac{1}{T_{obs}}\right)$$



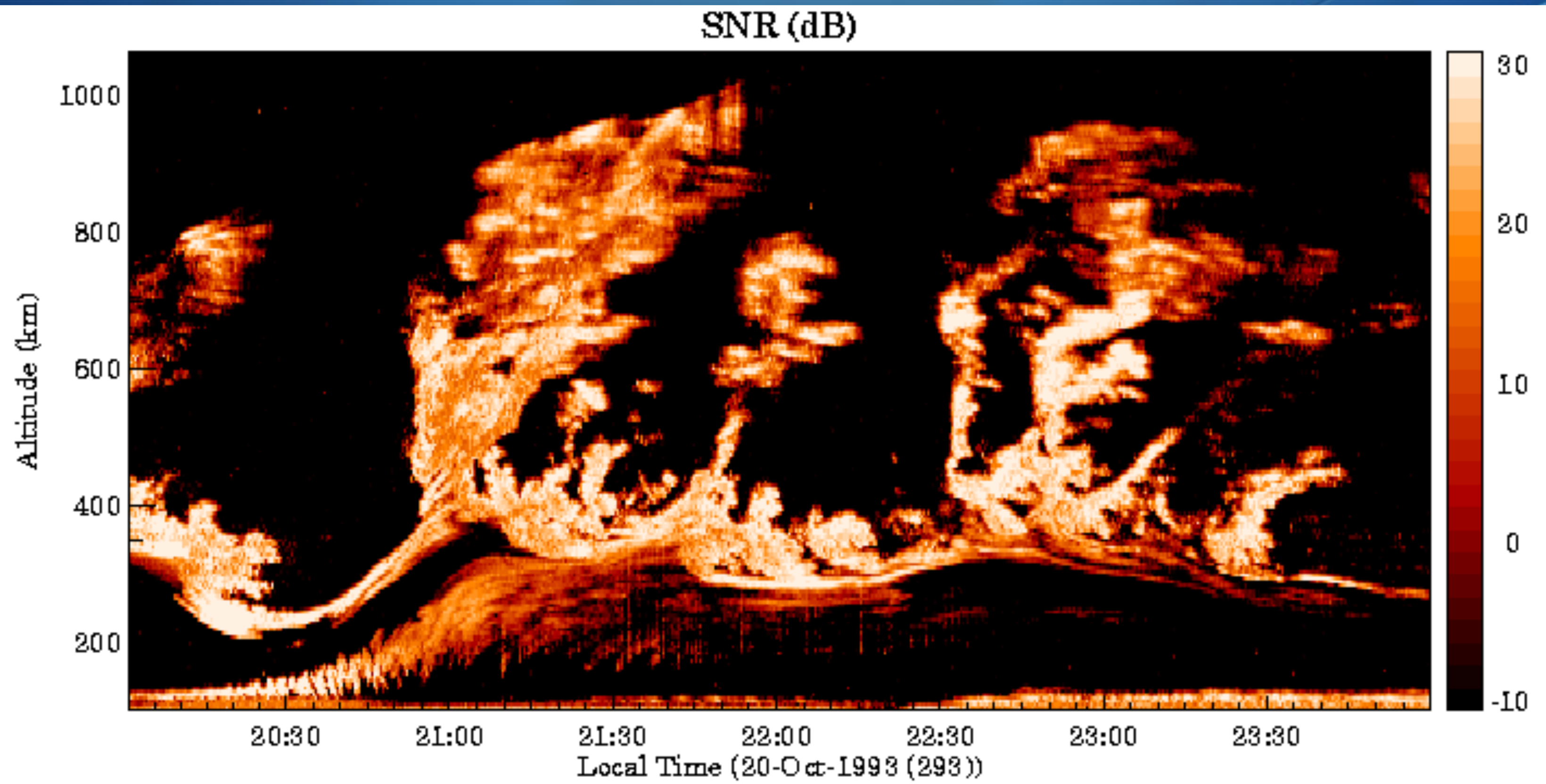
Interferometry at Jicamarca

- Measurement of magnetic field inclination
- Studies of field-aligned irregularities
  - Large scale waves (EEJ)
  - Zonal velocity measurements (EEJ, ESF)
  - Aspect sensitivity measurements (EEJ, 150-km, ESF)
- Meteor studies
  - Non-specular Meteor trails
  - Meteor heads

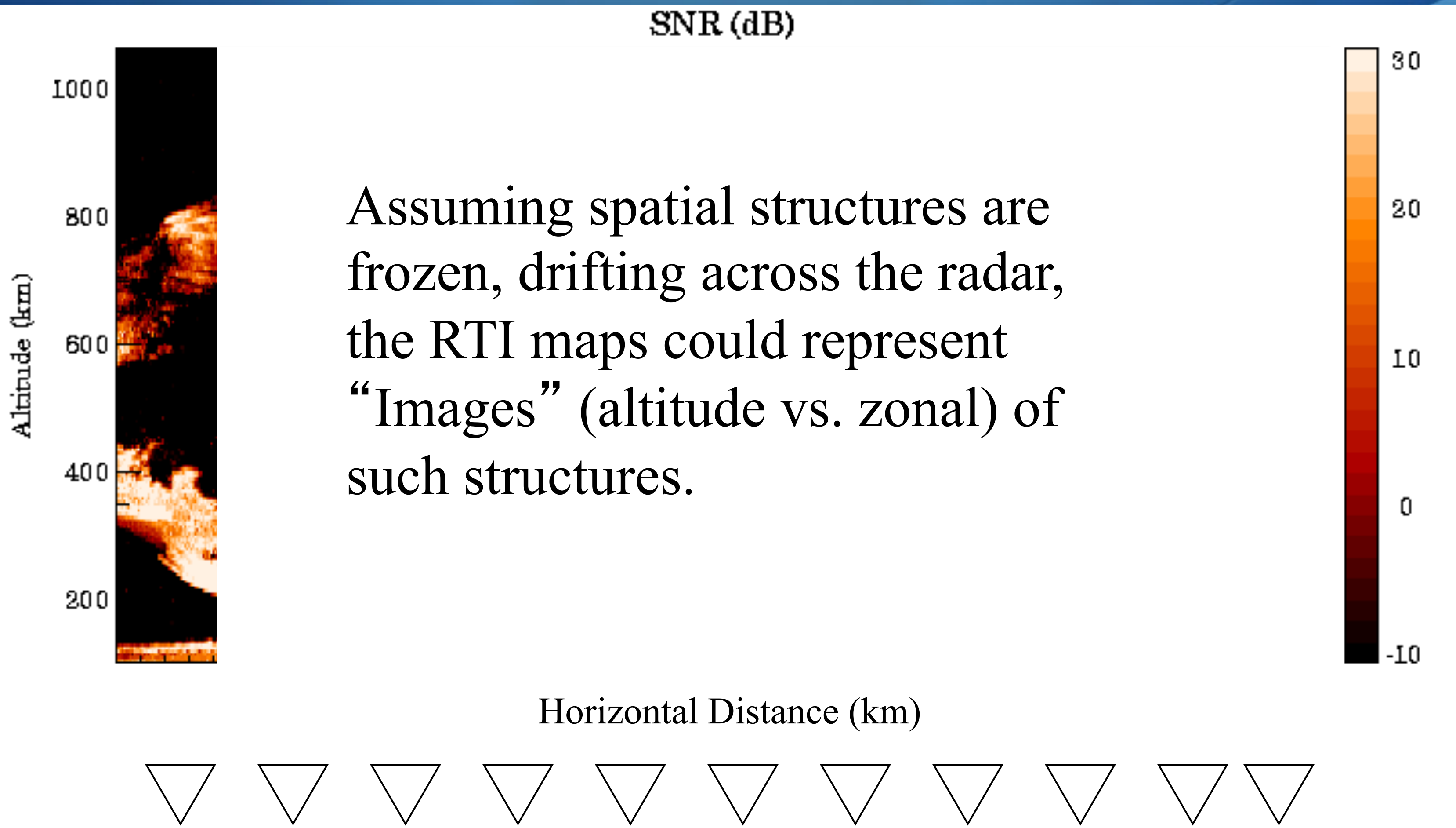




Range-Time Intensity (RTI) maps

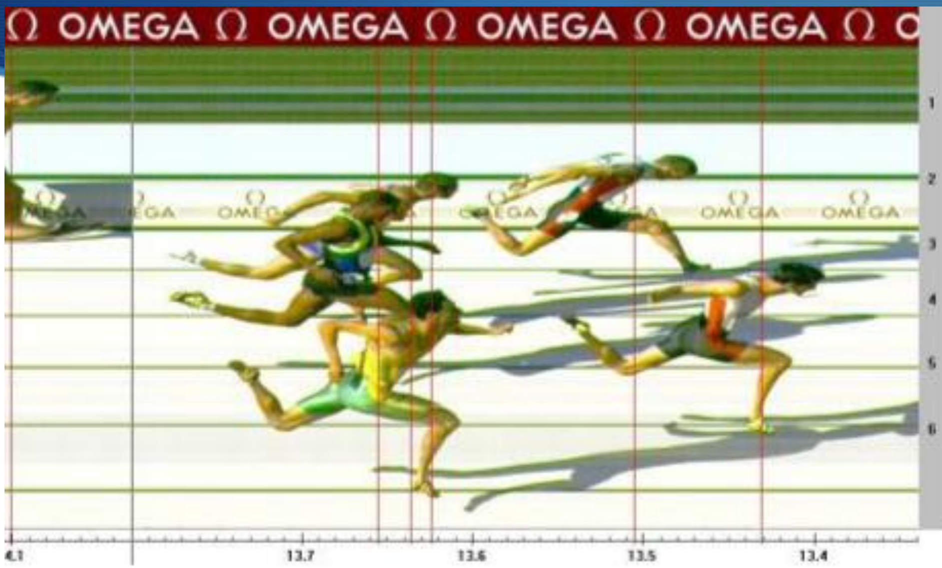


Range-Time Intensity plots as "Images": Slit camera interpretation



Assuming spatial structures are frozen, drifting across the radar, the RTI maps could represent "Images" (altitude vs. zonal) of such structures.

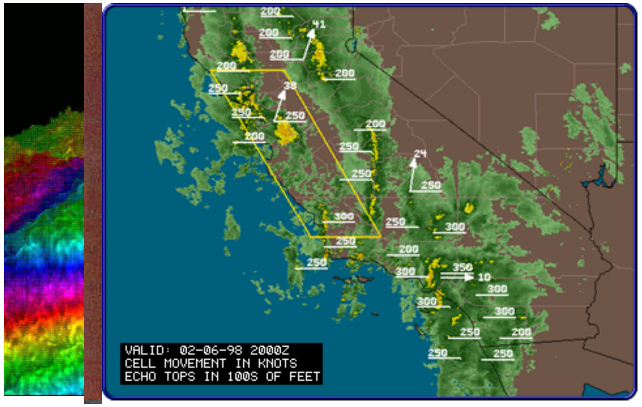
Slit-camera Analogy and Problems



- In some applications like races it is useful
- In many other applications it provides misleading results:
  - Slow structures are stretch out
  - Fast-moving structures are compressed.
  - In general, it is difficult to discriminate space-time features.

Imaging in other fields

- The radar imaging techniques used in studying the ionosphere, are an extension of more general techniques used in getting images with non-visible wavelengths (radio frequency, infrared, ultrasound, etc.) and in particular of imaging techniques with radars.



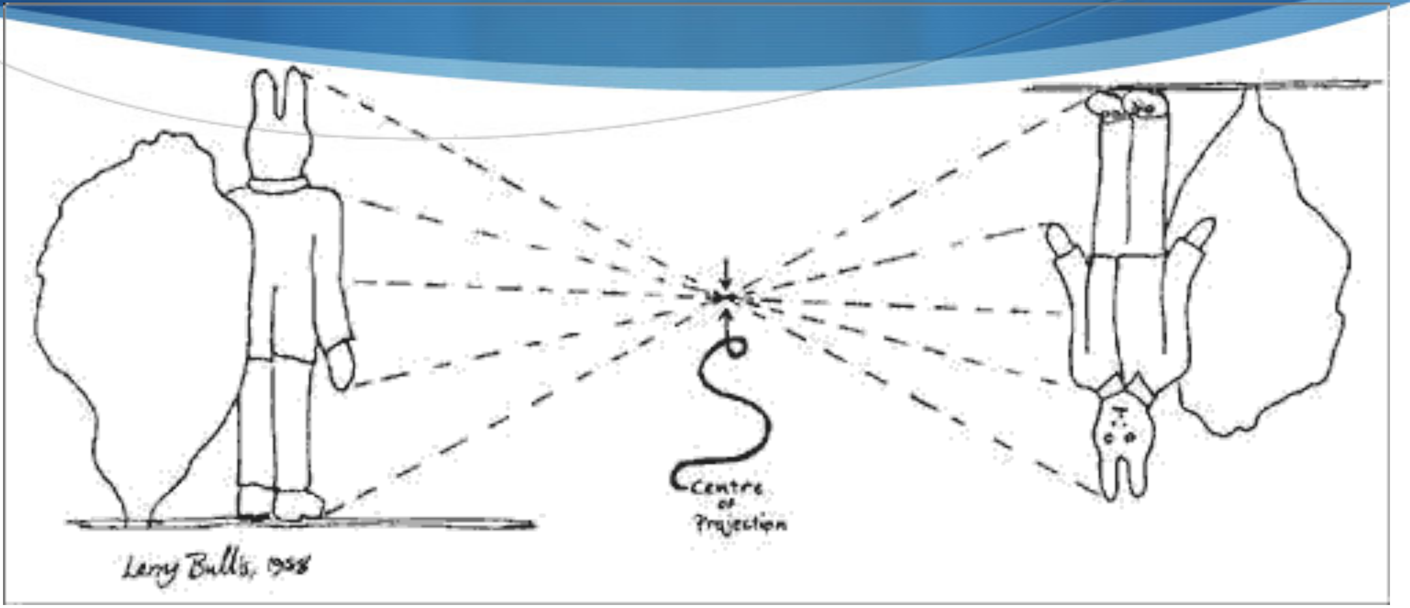
- Among radar imaging techniques we have:
  - SAR Imaging
  - Planetary imaging
  - Underground imaging (Georadar)
  - Meteorological radar images
  - Ionospheric images

- Radar imaging are part of more general techniques used in Astronomy.

Atmospheric/Ionospheric Targets

- In atmospheric applications, besides the problems of discriminating space-time features of moving structures across the beam, radars have finite beam widths and "irregularities" can appear, grow, dissipate, disappear, inside the beam!

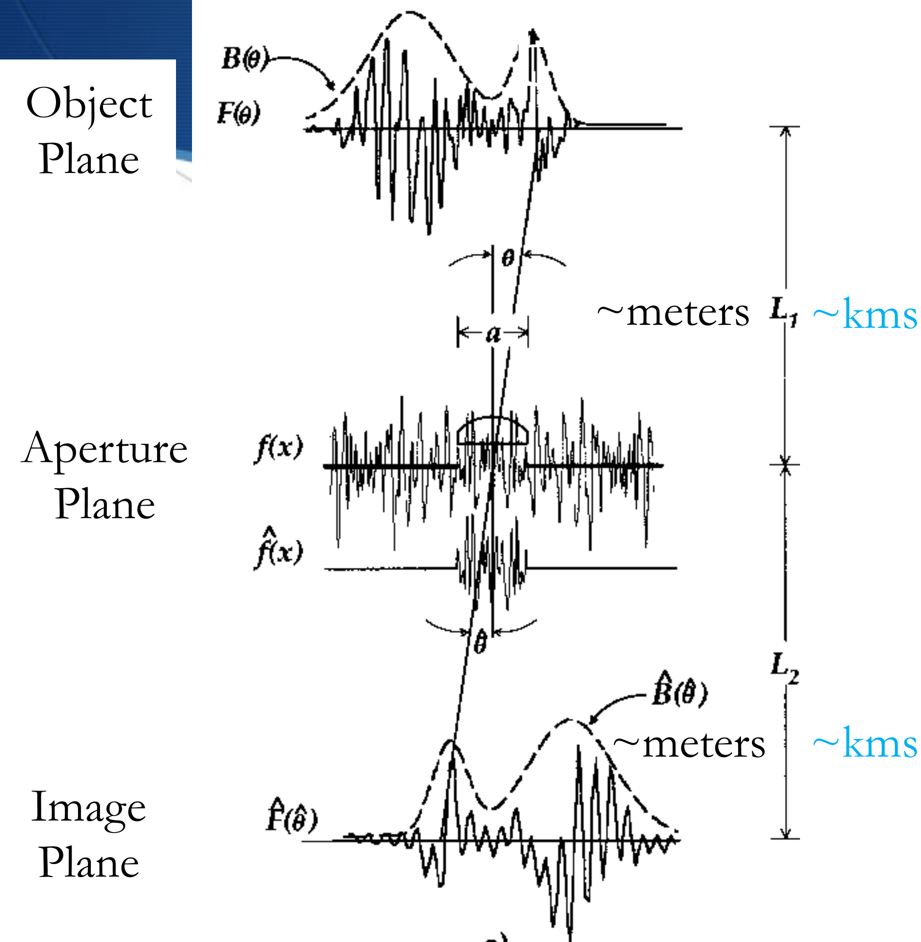
Analogy with an Optical Camera
The pin-hole camera



$$\text{Object} \underbrace{\qquad\qquad\qquad}_{\text{Far field}} \text{Aperture} \underbrace{\qquad\qquad\qquad}_{\text{Far field}} \text{Image}$$

Optical wavelength ~ 500 nm

Analogy with an Optical Camera



[from *Woodman*, 1997]

$B(\theta)$ True Brightness

$$B(\theta) \xleftrightarrow{Fourier} V(r)$$

$$\hat{B}(\theta) = A^2(\theta) * B(\theta)$$

$$\hat{B}(\theta) \xleftrightarrow{Fourier} \hat{V}(r)$$

$\hat{B}(\theta)$ Estimated Brightness

- In radio astronomy: Camera without "flash"
- In radar imaging: Camera with "flash"

BP Imaging!

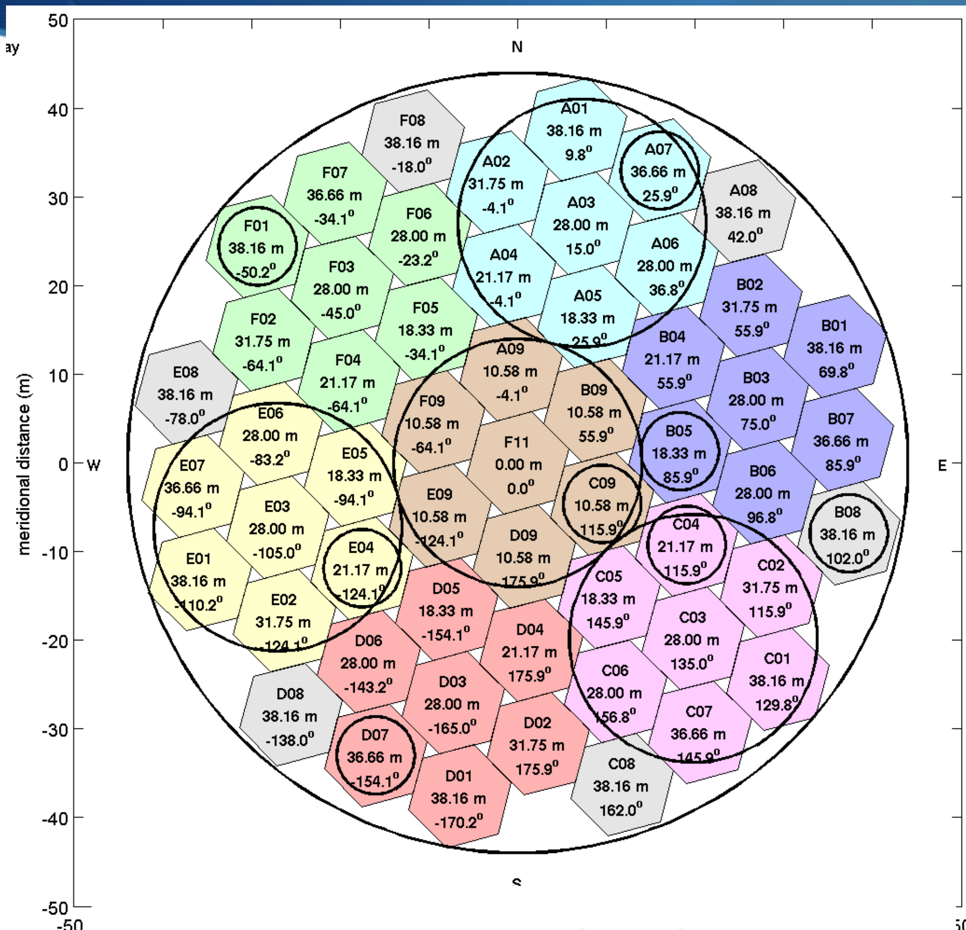
Aperture Synthesis Radar Problem

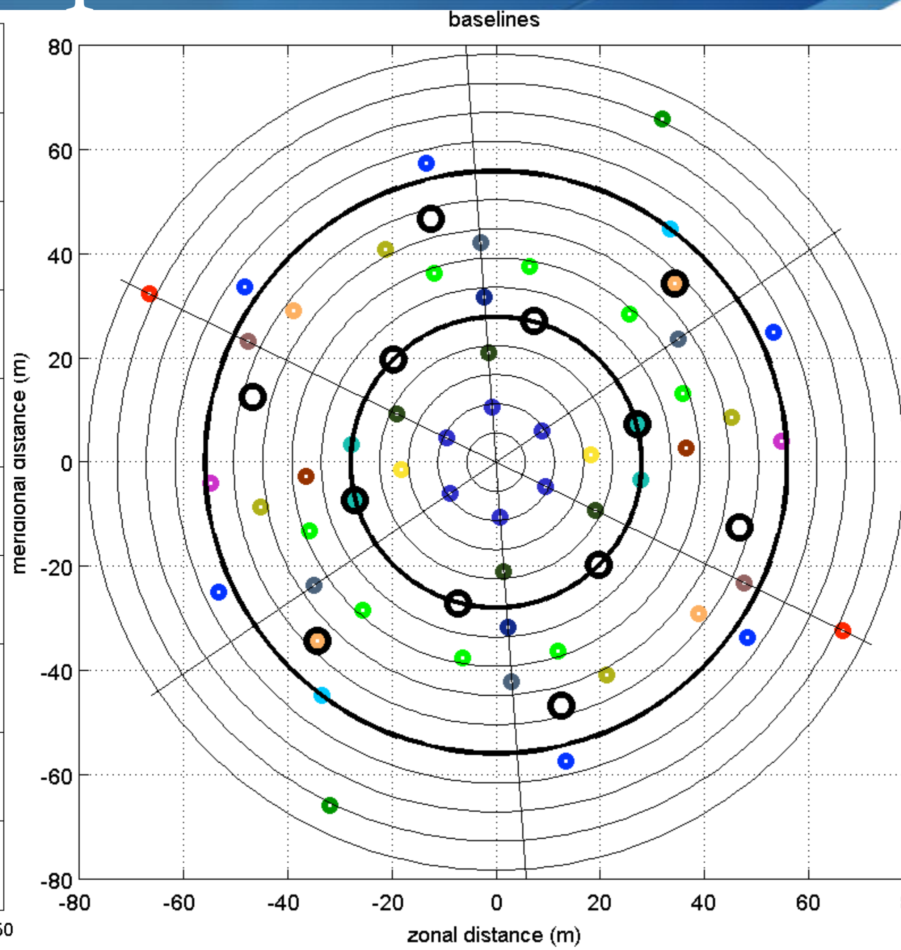
Given: $$\hat{V} = \mathbf{a} * \mathbf{a}^{*} \times \mathbf{M}\,\mathbf{B}$$

- find $\hat{B}$ that best agrees with B.
- where **a** can be an arbitrary complex vector, e.g.,
  - Truncation (like windowing in spectral analysis)
  - Gaps
  - Focusing (needed for near-field applications)

$$\mathrm{B}(\theta,\omega) = A(\theta)\mathrm{B}_{\mathrm{atm}}(\theta,\omega)$$

Rx Antennas vs Visibility: MAARSY Example





$$V(\mathbf{k}\mathbf{d}_j) = \frac{\langle v_1 v_2^* \rangle}{\sqrt{\langle |v_1|^2 \rangle - N_1}\sqrt{\langle |v_2|^2 \rangle - N_2}}$$

[Courtesy M. Zecha]

Spatial Spectral Estimation Problem

- The radar imaging problem reduces to locating the radiating sources (e.g., EEJ irregularities) by using an array of sensors (antennas). This type of problem finds applications in radar and sonar systems, communications, astrophysics, biomedical research, seismology, underwater surveillance, …

- Since the problem basically consists of determining how the "energy" is distributed over space, with the source positions representing points in space with high concentrations of energy, it can be named a *spatial spectral estimation problem*.

- A main location of a radiating source in 3D space is defined by three parameters, namely range (r), azimuth and elevation. The range is often measured by means of return time of travel in active systems (e.g., radar) and by means of time delays measured at a number of sensors in passive systems (e.g., radio astronomy). The azimuth and elevation angles are obtained from the measurements of the direction of arrival (DOA) or angle of arrival (AOA). The "width" of the source can be obtained from the coherence of the signal between sensors.

- The wave field generated by the sources travels through space and is sampled in both space and time by the sensor array. By making analogy with the temporal sampling, we may expect that the spatial sampling done by the array provides more and more information on the incoming waves as the array's aperture increases.

Considerations in Atmospheric/ Radar Imaging

- In atmospheric/ionosperic radar imaging, scattered signals are caused by refractive index fluctuations on the propagation path of transmitted radar pulses.

- At VHF and higher frequencies the scattered signal fraction is so minor that there is no need to consider secondary scattering of the scattered signals or the extinction of a propagating pulse due to scattered energy.

- From Radio Astronomy jargon, we called:
  - *Brightness*, the angular distribution (θ x, θ y) of the target we are after (e.g., EEJ, ESF) multiplied by the transmitting and receiving antenna patterns.
  - *Visibility*, the spatial autocorrelation function of the field at the aperture, due to the *Brightness*. It is a function of distance in x and y (rx,ry).

- *Brightness* and *Visibility* form Fourier Transform pairs.

- From analogy with temporal-frequency spectral estimation:
  - The largest separation between antennas, is equivalent to the observation time. Therefore it determines the angular resolution.
  - The shortest separation between visibility samples, is equivalent to temporal sampling period, and determines the maximum unambiguous angular range that can be imaged ("Nyquist angle").

Ionospheric Radar Imaging: Peculiarities of the target

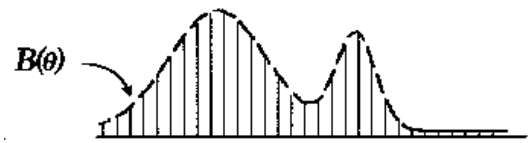
- Spatially 3D.
- Changes with time in two different time scales:
  - short, that defines the "color" (frequency spectrum)
  - long, non-stationary scale.
- It is non stationary and non homogeneous, representing a stochastic process of 4 dimensions.
- Normally, once has a small number of independent samples to average.
- Given these characteristics, how do we solve the problem?
  - Shall we do a simple Fourier inversion?
  - Are there any constraints or a priori information that we could consider? If so, how?

Aperture Synthesis Radar Imaging Algorithms

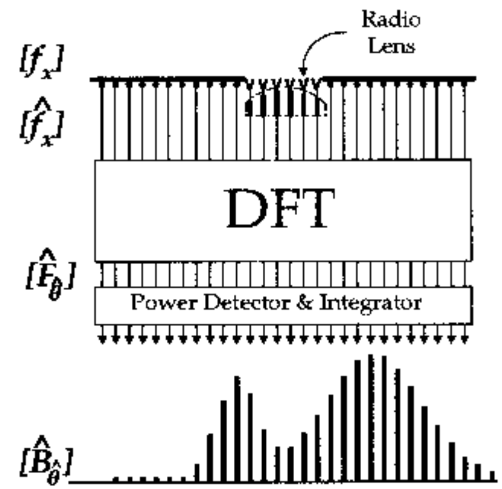
- Beamforming or Non-parametric Methods. These methods do not make any assumption on the covariance structure of the data.
  - Fourier-based
  - Capon or Linear Constraint Minimum variance beam forming
- Parametric methods. These methods usually make use of a known functional form B, error covariances, or other *a priori* information.
  - **Maximum Entropy**
  - Model fitting. For example, a Brightness modeled by N anisotropic Gaussian blobs
  - Single value decomposition (SVD)+MUltiple SIgnal Classification (MUSIC) (developed for the localization of discrete scatterers)
- Other
  - CLEAN, RELAX (Super CLEAN) (from Radio Astronomy)
  - APES, variants of Capon, etc. (from high resolution spectral analysis)

Fourier-based Methods

$$\hat{\mathbf{V}} = \mathbf{a} * \mathbf{a}^* \times \mathbf{MB}$$



- Ideally **B** could be obtained by a simple Fourier inversion.
- If **a** only represents truncation, i.e., **V** is not zero outside the aperture, the estimation can be improved by using proper weighting functions (equivalent to windowing in spectral analysis). The use of weighting functions is also called as *apodization*.
- If **a** represents near field effects, e.g., for lower atmospheric applications, the image can be *focused* by correcting it with a known **a**.
- If **a** is more complicated, which is usually the case (e.g., representing gaps, truncation), clever approaches are needed to "CLEAN/deconvolve" dirty brightness images obtained from Fourier inverting.



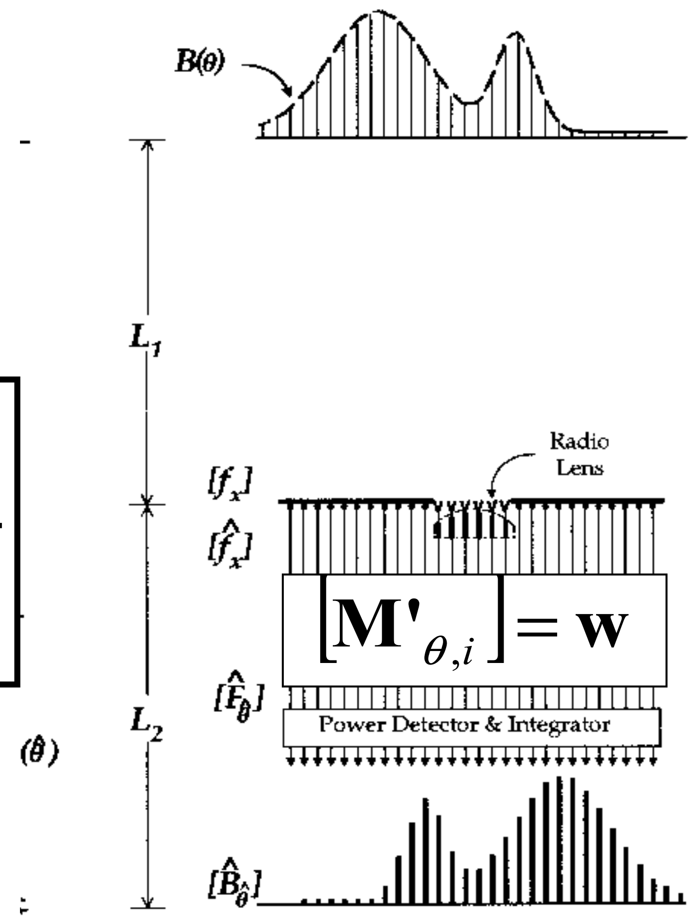
Capon Method

$$\hat{B}_{\text{capon}}(\theta,\omega)=\mathbf{w}^{+}\hat{\mathbf{V}}(\omega)\mathbf{w}$$

$w(\theta)$ is such that $\hat{B}_{\text{capon}}(\theta,\omega)$ is a minimum

under the constrain $\sum w_i(\theta)\text{plane}(\theta,i)=1$

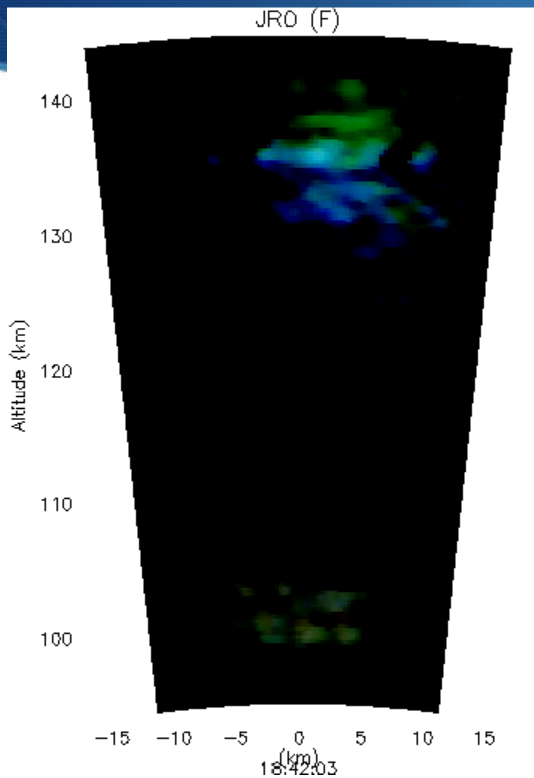
$$\boxed{\hat{B}_{\text{capon}}(\theta,\omega)=\frac{1}{\mathbf{e}^{+}\hat{\mathbf{V}}(\omega)^{-1}\mathbf{e}}}$$

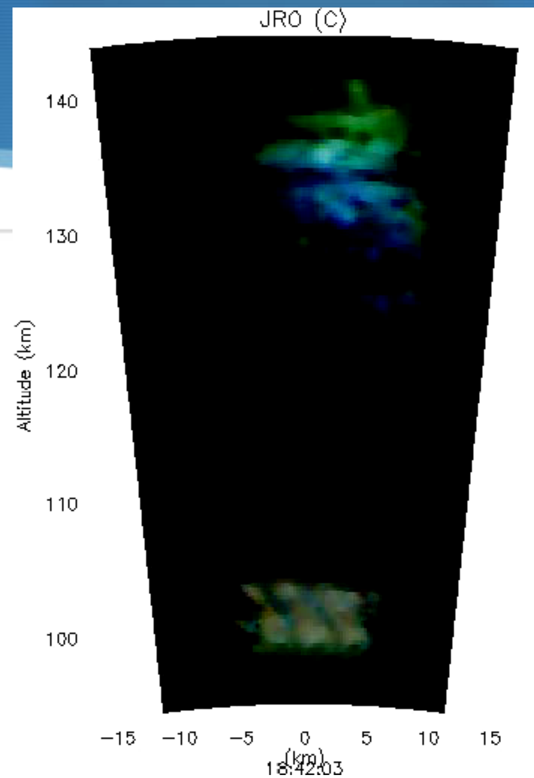


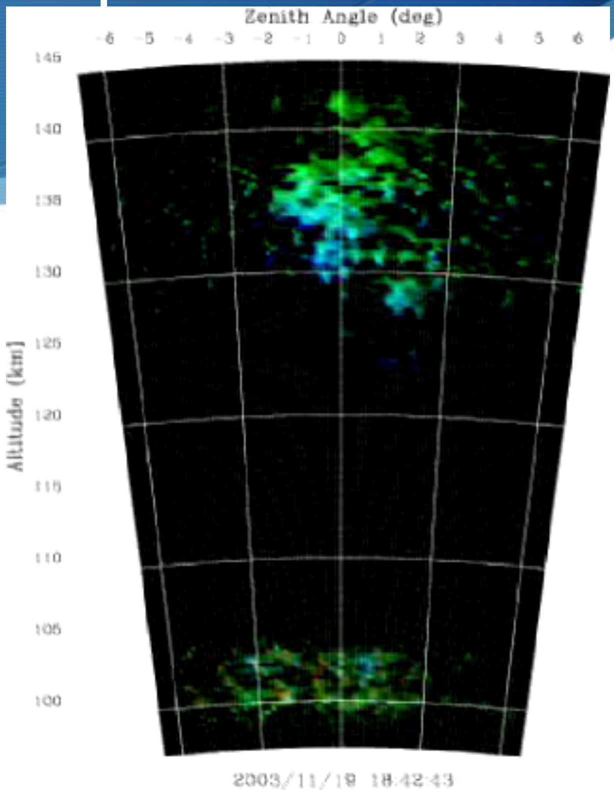
- It reduces to a matrix inversion!!!!
- It is an adaptive method that finds a suitable *w*, based on the data for each *pointing* angle.

[for more details, talk by Marcial on Friday]

Comparison of Aperture Synthesis Radar Imaging Techniques





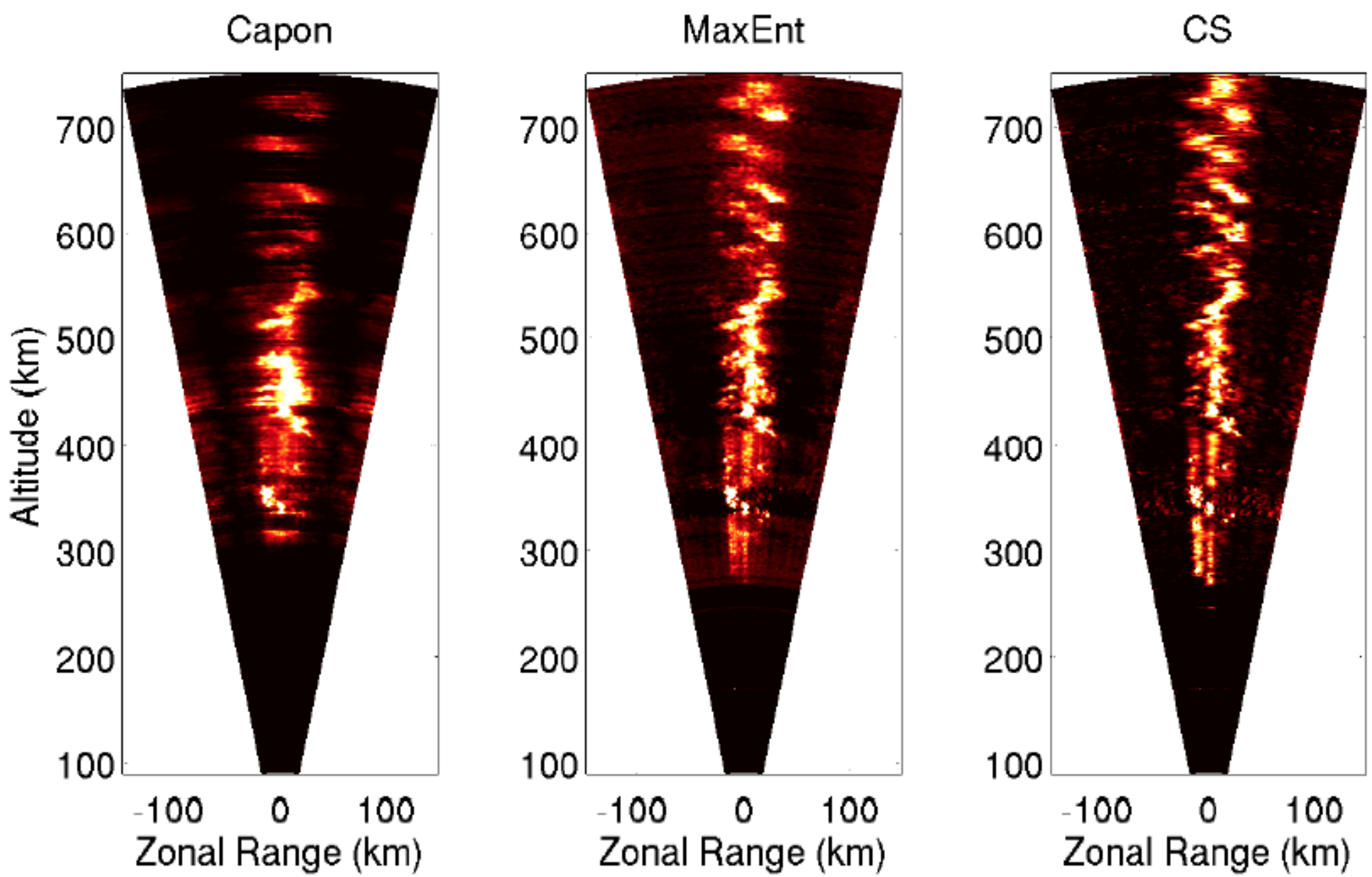


- Very Easy to implement
- Very fast processing
- Poor resolution.
- Requires good SNR.
- Usually gets “dirty” images
- Fourier Transform

- Easy to implement
- Fast processing
- Adaptive pointing
- Requires good SNR
- Matrix inversion

- Hard to implement
- Slow processing
- Better estimates and Higher resolution.
- Takes into account statistical errors
- Set of non-linear equations

More comparisons: Capon, MaxEnt and Compressed Sensing



[from *Harding and Milla*, 2013]

Maximum Entropy Method (MaxEnt)

- Model-based inversion process where $B(\theta,\omega)$ is a positive definite function constrained to be consistent with the data ($\hat{V}(\mathbf{r},\omega)$) to a specified accuracy (based on self and statistical errors).

$$\hat{B}(\theta,\omega) \propto \frac{e^{-\lambda_j h_j(\theta)}}{\int e^{-\lambda_j h_j(\theta)} d\theta}$$

λ_j : Lagrange multiplier, one for each visibility measurement

$h_j(\theta)$: point-spread function of the interferometer

denominator: like the Gibbs canonical partition function

- By maximizing the Shannon Entropy ($S \propto B_j \log B_j$), including the constraints, one gets a well-posed set of N simultaneous non-linear equations for N Lagrange multipliers.
- The estimation problem is reduced to solve numerically the set of non-linear equations (e.g., Powell method).
- It is a method that "deconvolves" via regularization.

Model Fitting

- In this technique one makes a parametric model of **B** and uses imaging equations to calculate the expected measurements. One then adjust the parameters of the model to get the "best fit" model.

- There are three steps involve in model fitting: (1) design a model defined by a number of adjustable parameters; (2) Choose a *figure-of-merit* function; (3) Adjust the parameters to *minimize the merit function*.

- The goals are to obtain: (1) Best-fit values for the parameters, (2) A measure of the goodness-of-fit of the optimized model (relative to the measurement errors); (3) Estimates of the uncertainty in the best-fit parameters.

- Model fitting has many desirable properties. It can take into account all the details of the measurement process (e.g., antenna patterns, calibration phases), which won't be a simple Fourier transform; and because it operates in the domain of observation **(V),** where the statistics of the measurement process are well understood (error covariances), it can estimate the statistical uncertainties of the parameters of the best-fit model, which may be the atmospheric/ionospheric important quantities.

- However, it also has serious problems: it may be difficult to choose a suitable parameterization model, the solutions are not unique, and it can be much, much slower than the conventional beamforming and deconvolution methods.

Uses an practical considerations of model fitting

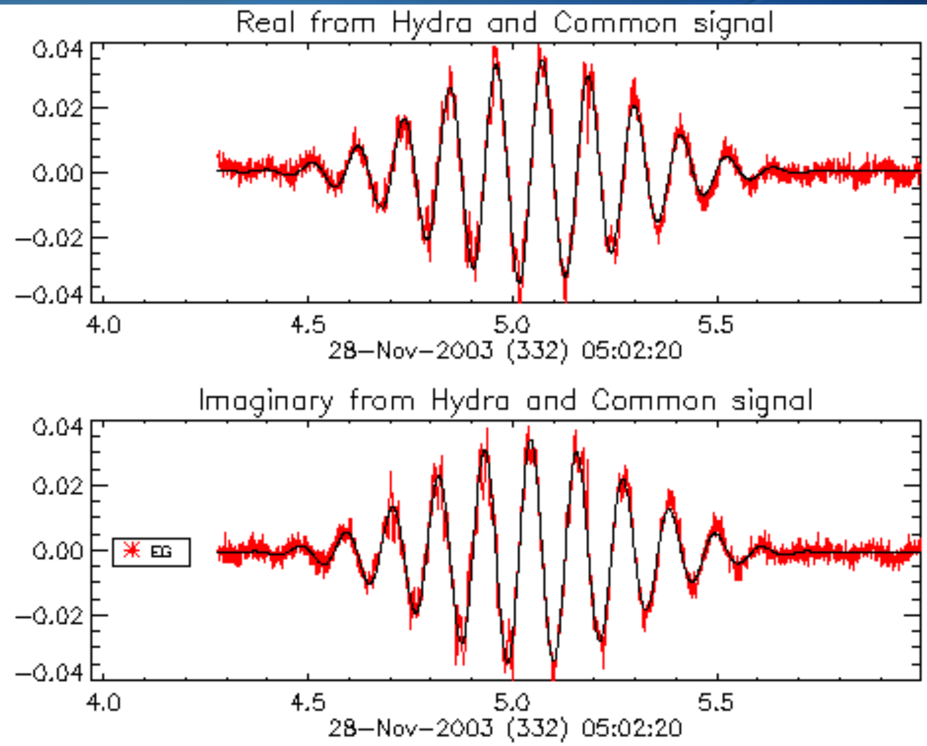
- If the primary objective is the image, then model fitting is not useful, it is better to use conventional methods. However, model fitting is very useful for interpreting sparse or uncalibrated data, and for quantitative analysis.

- Model fitting could be used for the following:
  - Checking and adjusting amplitude and/or phase calibration against known sources, e.g., modeling a Radio Star as a point-source.
  - If the source is simple enough to be accurately represented by a parametric model, then model fitting provides accurate estimates of the model parameters. Estimates derive in this way directly from **V** will always be more reliable than parameters estimated from a deconvolved image.
  - Model fitting can be used to improve the behavior of commonly used imaging techniques, e.g., CLEAN and Maxim Entropy.
  - Hybrid methods. For example, one can get an image using conventional images, and improve the estimate using model fitting. The conventional image would serve to provide a very good initial guess and more importantly a good idea of the model to be used, e.g., number of Gaussian blobs.

- Common fitting problems: (1) Finding global minimum; (2) Slow convergence; (3) Constraints; (4) Choosing the right number of parameters or components.

Phase Calibration Procedures

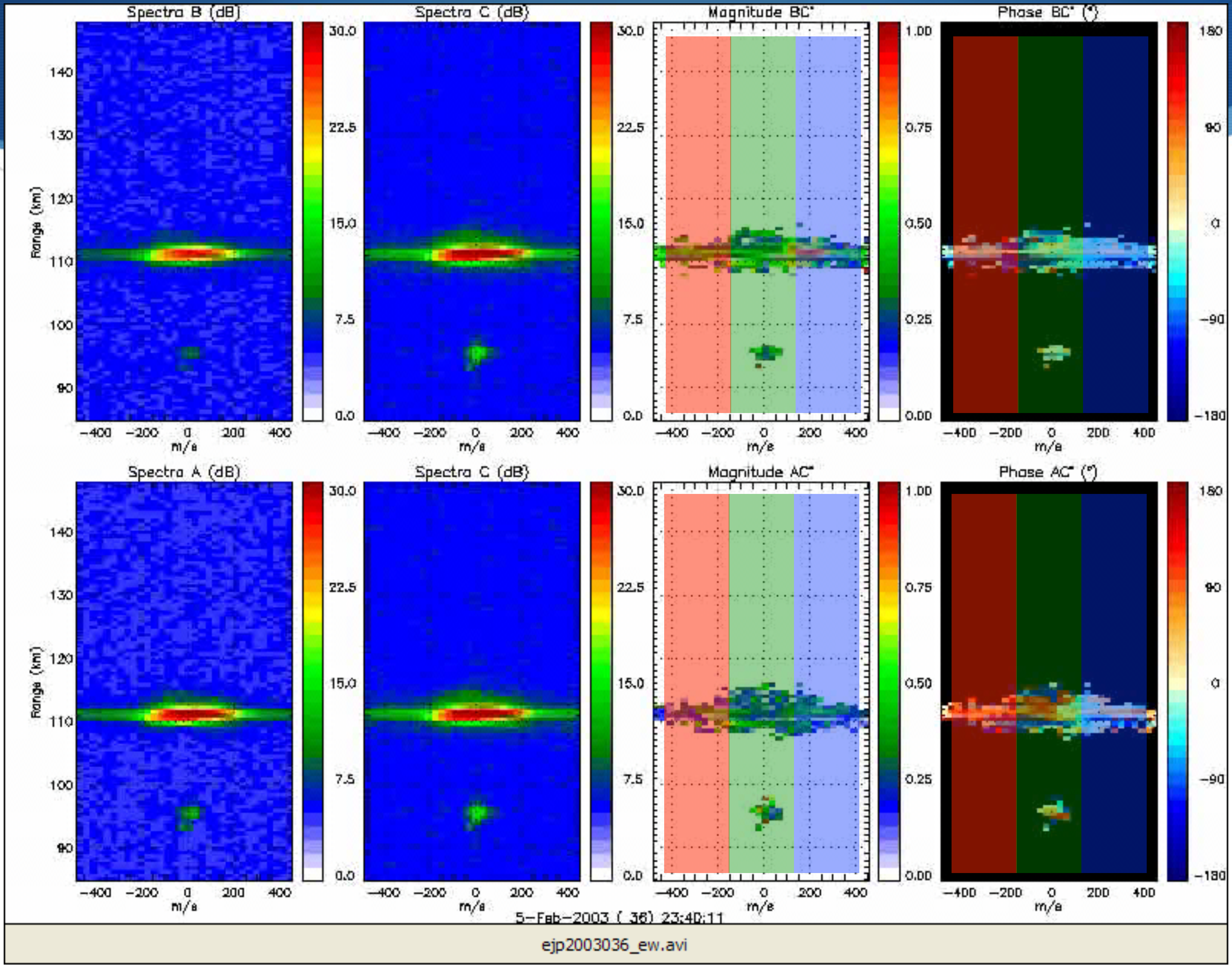
- Use of common signal
  - From natural sources.
    - Radio Stars [e.g., Cygnus A, Hydra].
    - Equatorial electrojet
    - Atmospheric signal after a "long" integration.
    - Meteor head echoes
  - From artificial sources
    - Mobile beacons with a GPS system
    - A common signal fed to receiving li [e.g., *Kudeki and Sürücü*, 1991].



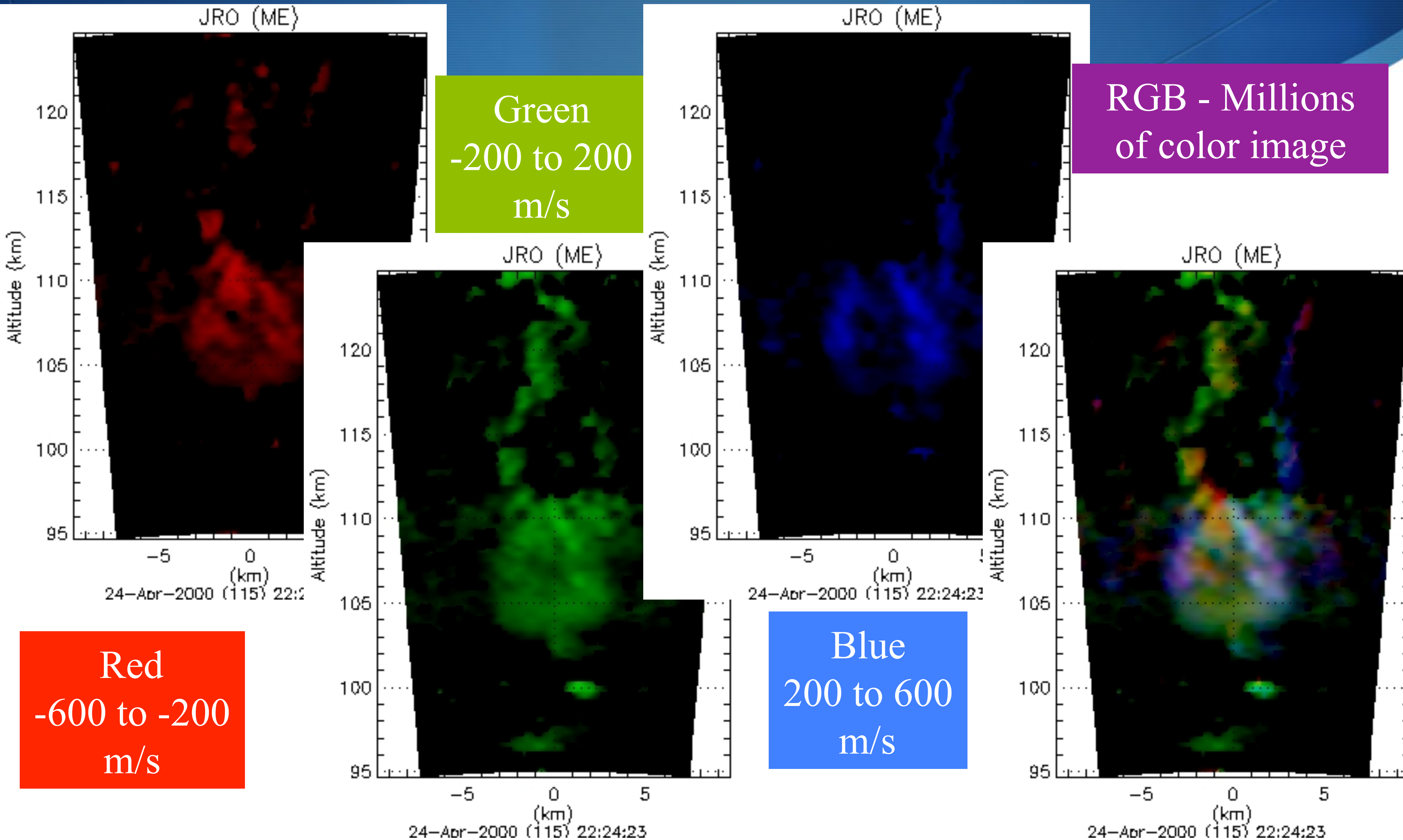
- "Optimizing" the estimated image
  - Maximize the entropy
  - Minimize peaks

$$\rho_H(t)_{ij} = A(0)_{ij} P_i^{1/2}(\theta_x,\theta_y) P_j^{1/2}(\theta_x,\theta_y) e^{i\frac{2\pi}{\lambda}(dx_{ij}\theta_x + dy_{ij}\theta_y) + iA(1)_{ij}} + A(2)_{ij} + iA(3)_{ij}$$

How to display radar images (2)



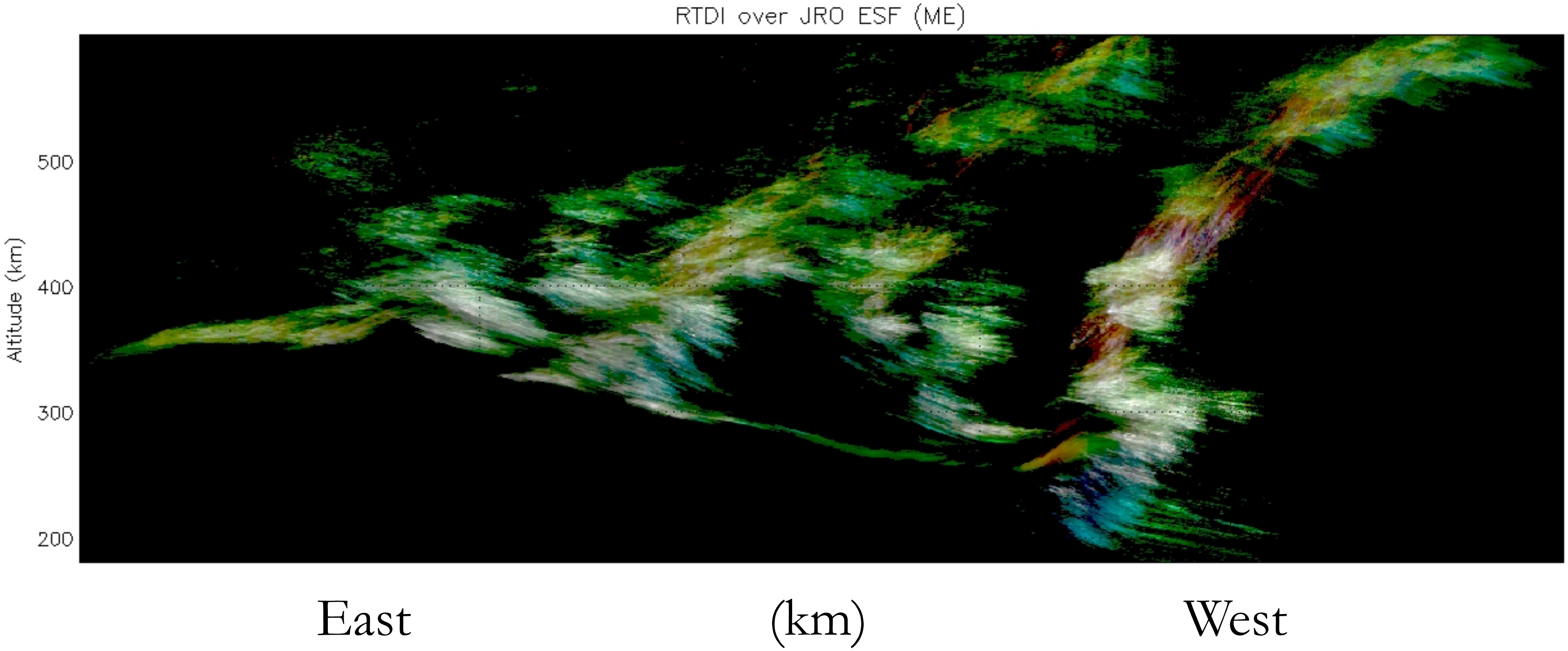
How to display radar images (3)



Examples of ESF Radar Imaging at Jicamarca

- Tx using two quarter antennas, phased to have a wide beam in the EW direction.
- 8 digital Rx channels for "imaging". A pair of modules can be used for single baseline interferometry.
- Automated phase calibration procedure, using beacon on the hill (relative). Absolute calibration from Hydra, meteor-heads, …
- 16-32 "colors" (FFT points)
- ESF images are obtained every 2 seconds and 300 m. The angular resolution is ~0.1-0.2°

ESF RTDI: Slit camera interpretation

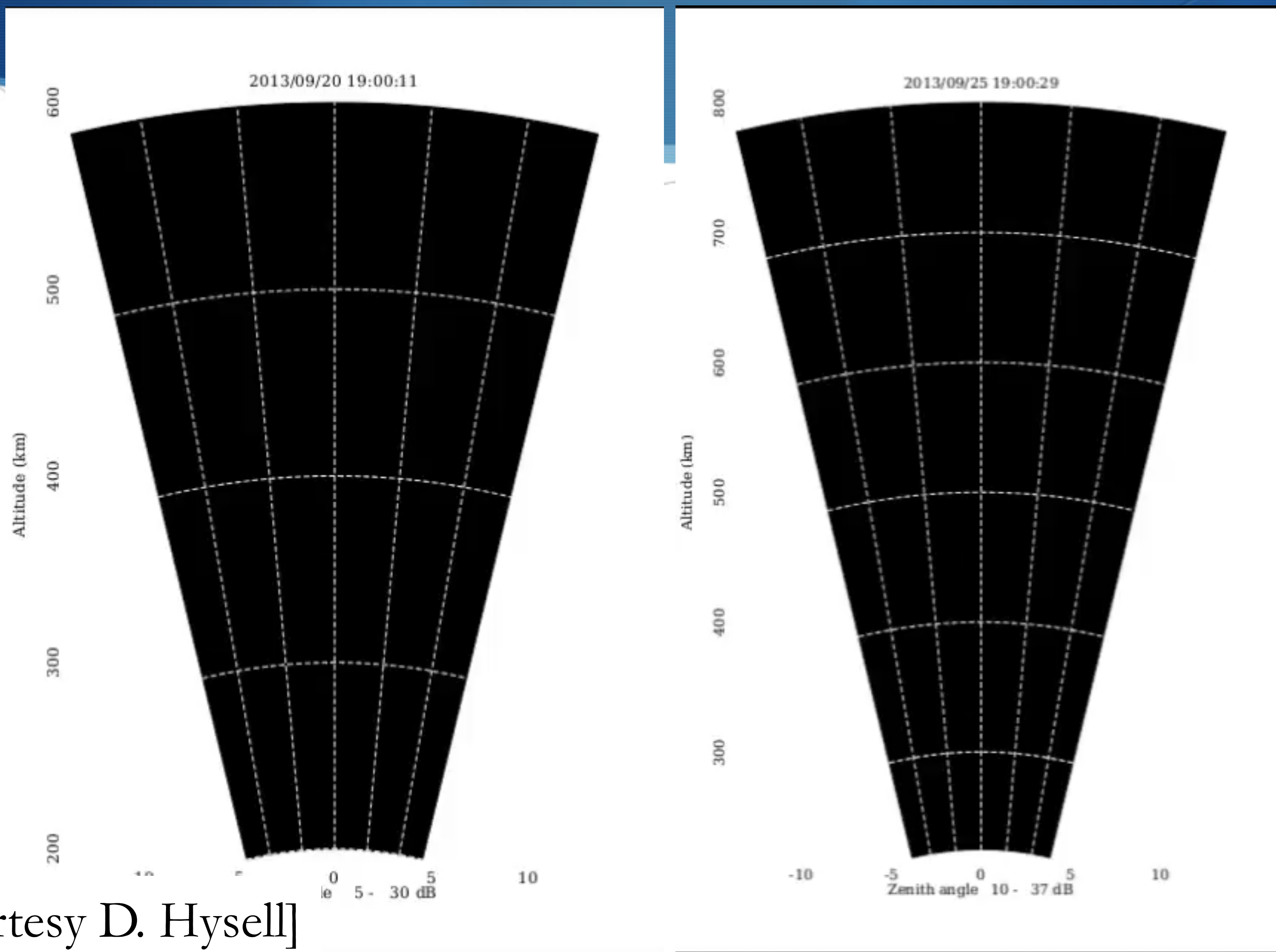


- Typical RTI maps are shown with "false" colors (colors from a pre-defined color table are associated to the signal intensity).

- Here we use Doppler for color. True 24-bit color range time intensity (RTI) plot using Doppler information (RTDI). RTI map is obtained for three Doppler regions centered around: -ve (Red), zero (Green), and +ve (Blue) Doppler velocities.

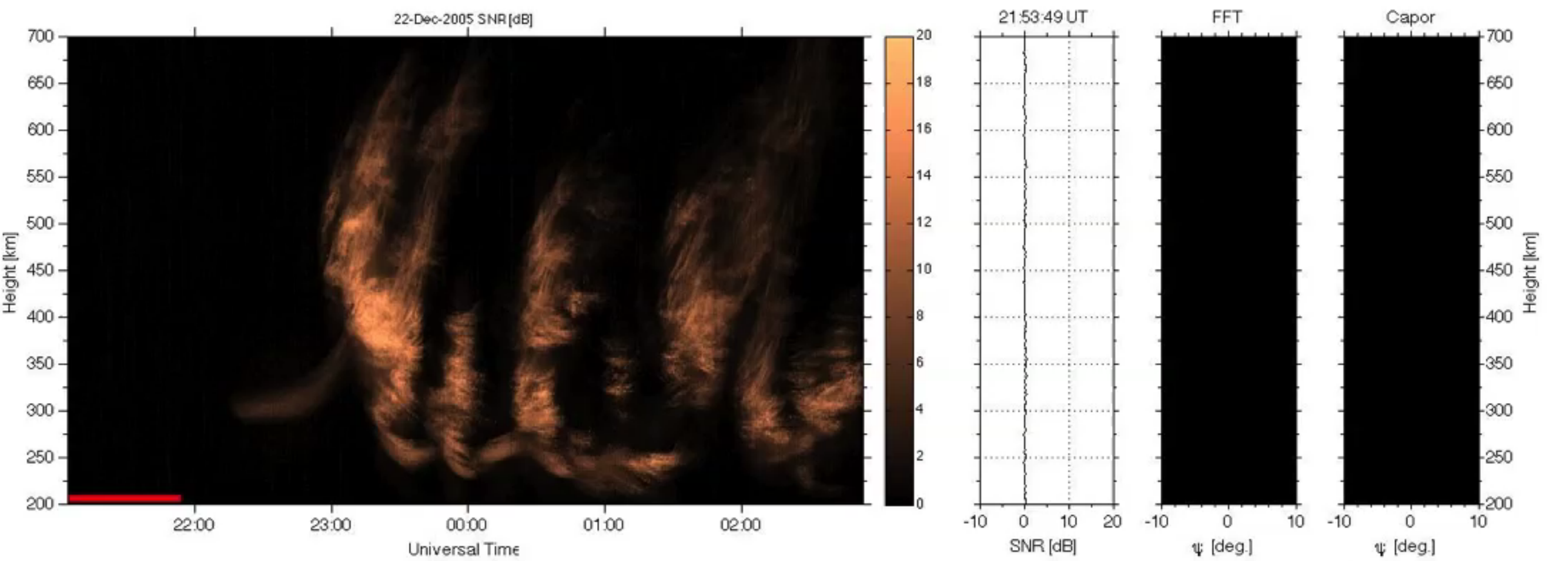
- It allows, for example, identification of regions and times where there is a depletion channel pinching off, Doppler aliasing, Doppler widening, etc.

Radio Movies of Ionospheric ESF Irregularities



[courtesy D. Hysell]

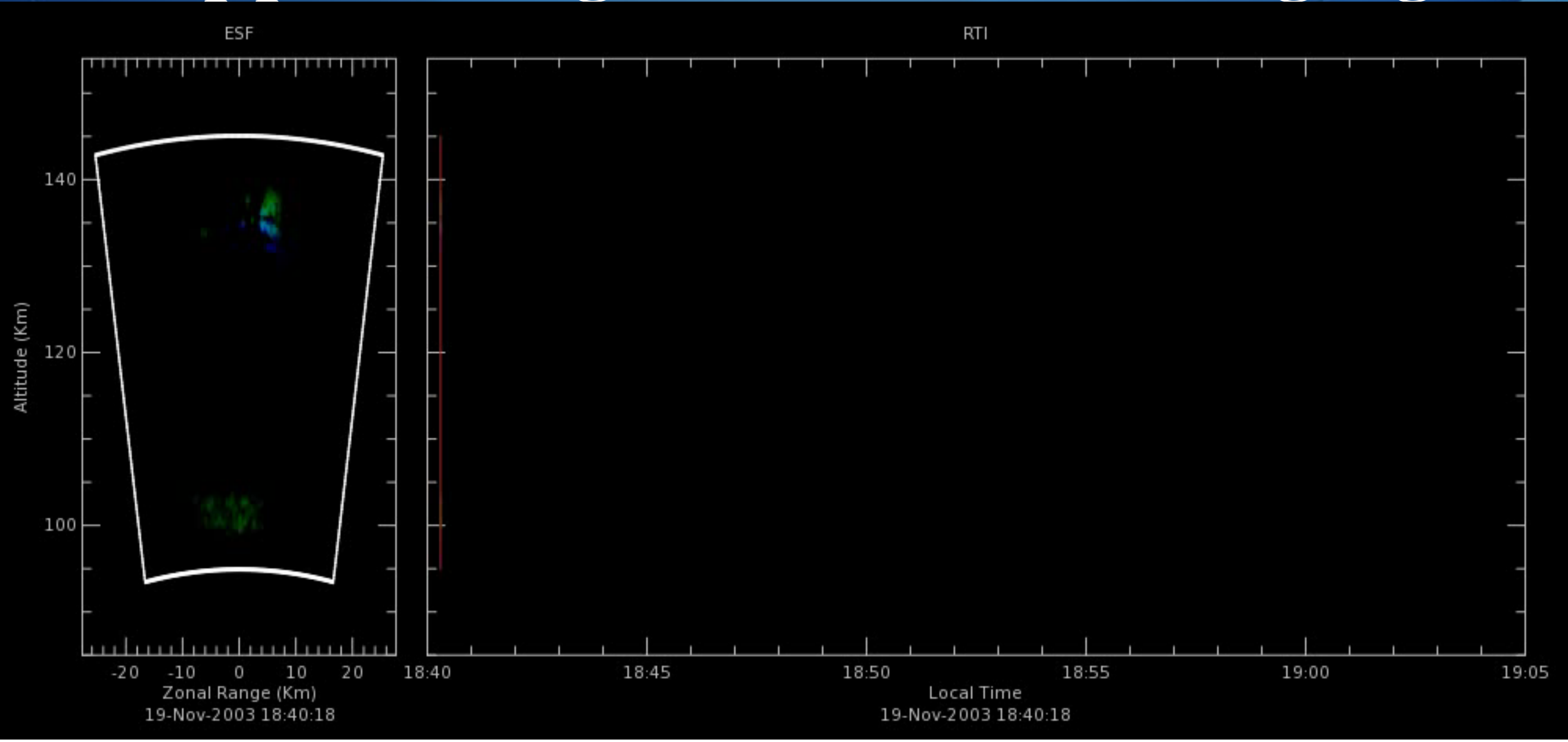
ESF example from Sao Luis



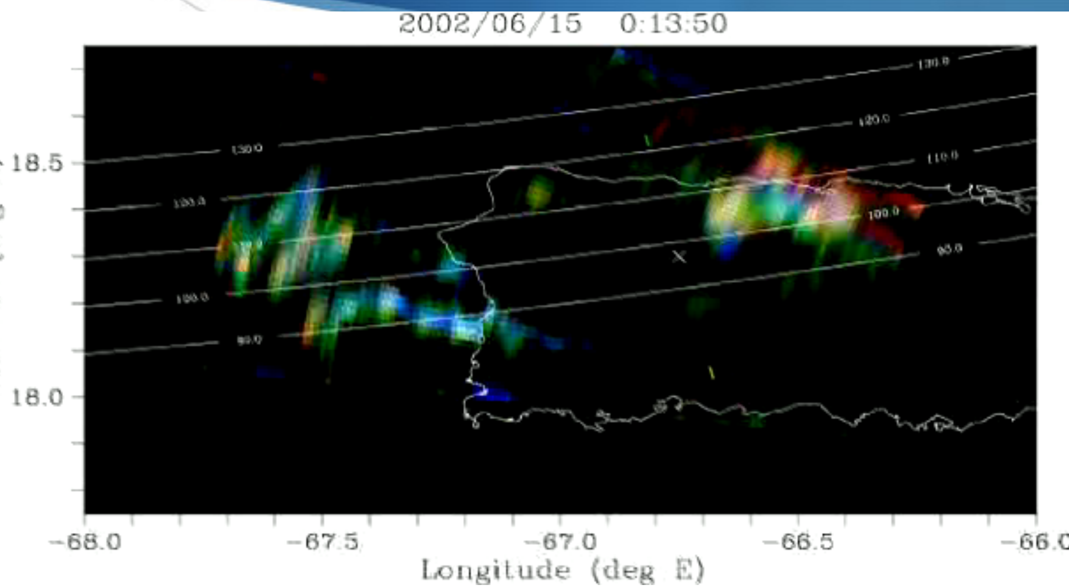
4 Rxs, 8 kW

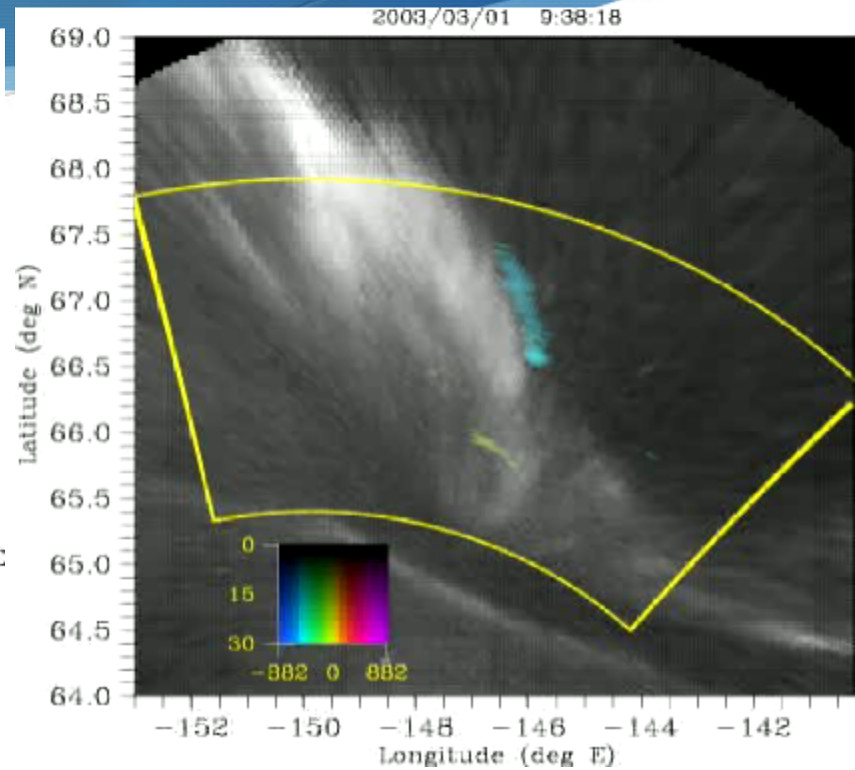
[courtesy F. Rodrigues]

Upper E region RTDI + Imaging



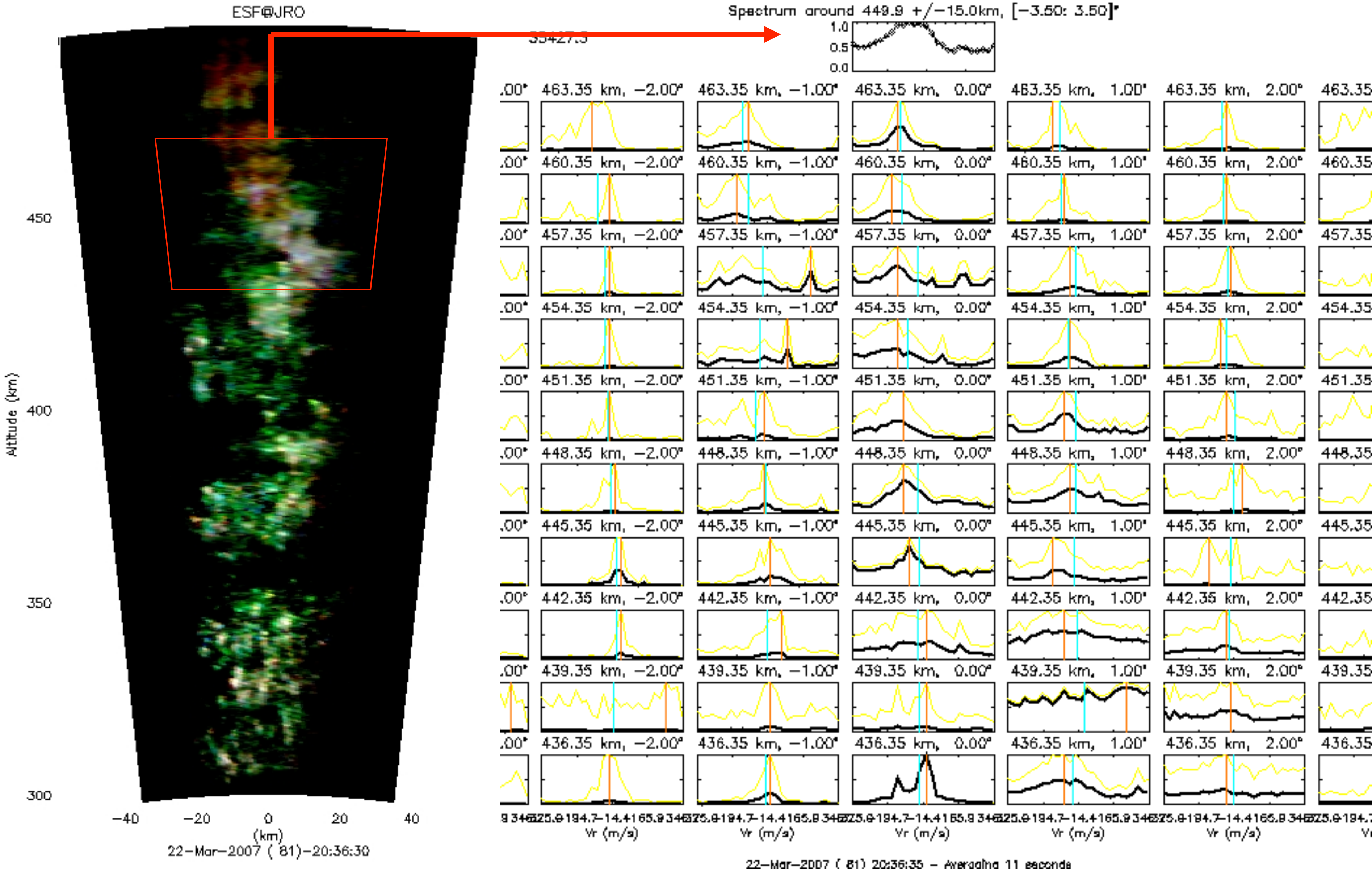
Radio Movies of Ionospheric Irregularities Mid latitude E region and Radio "Aurora"



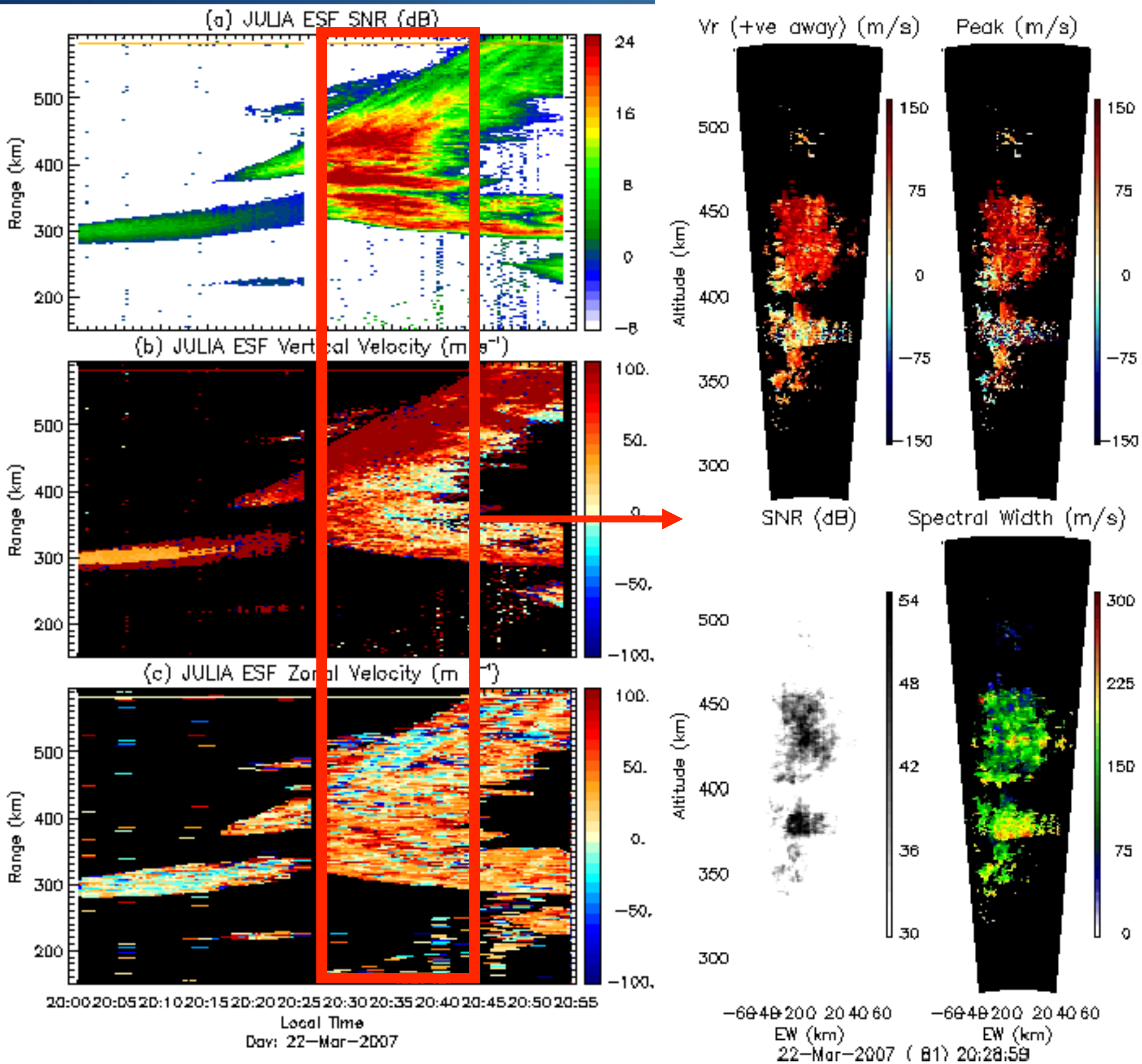


[courtesy D. Hysell]

“JP” Imaging Mode (1)



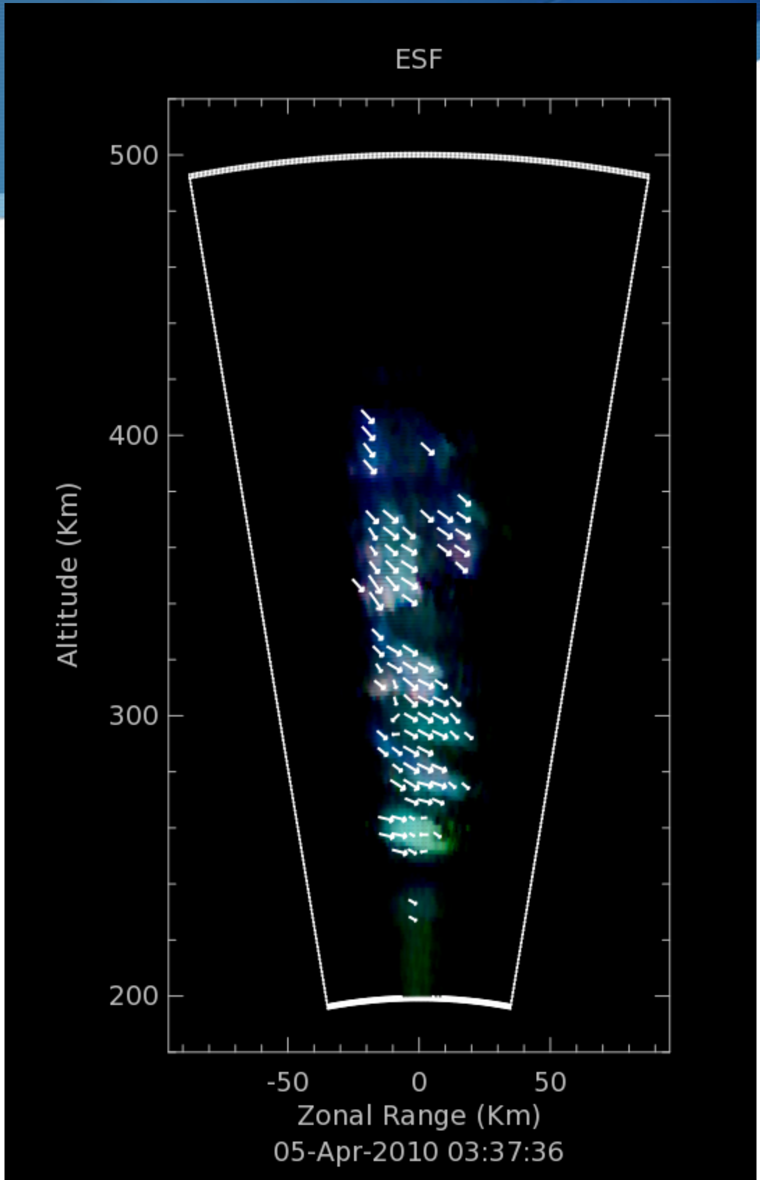
“JP” Imaging Mode (2)



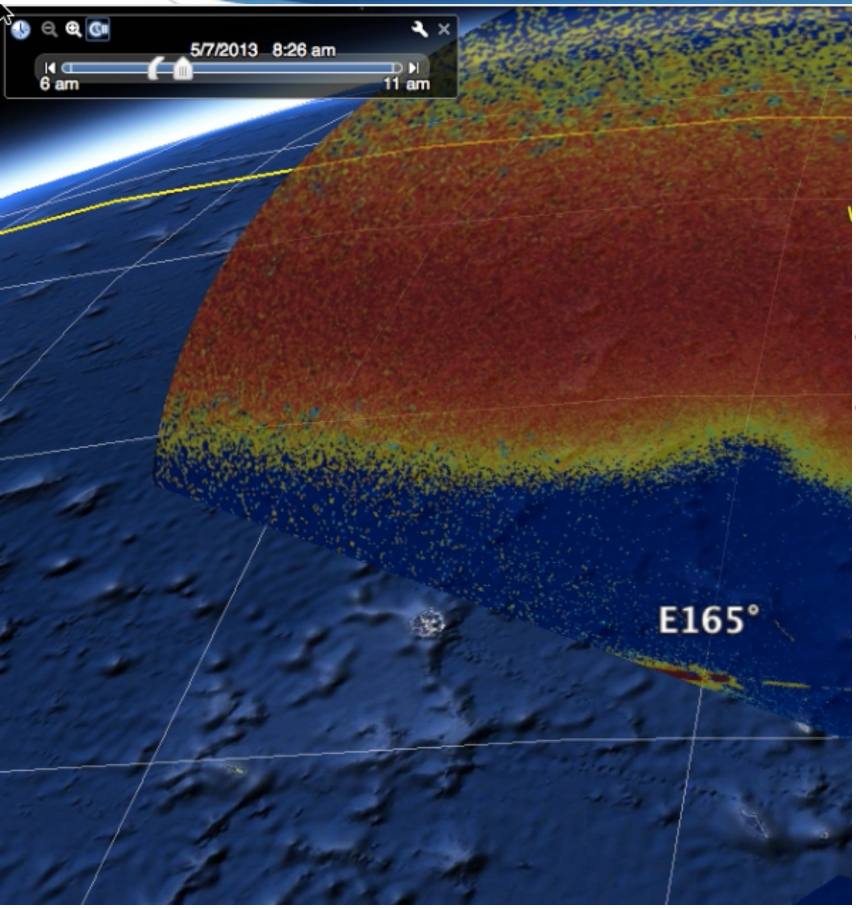
- JULIA-like parameters can be obtained from interferometry using a pair of antennas (SNR, mean vertical Doppler, zonal drift).
- Using imaging, we can get the spectra for different synthesized beams and range resolutions. In this example, each synthesized “scattering volume” is 0.2° and 600m obtained every 10 sec.
- Investigate the possibility of estimating the irregularity spectrum by measuring the radar Doppler spectrum with different averaging volumes [e.g., *Hysell and Chau*, 2004]

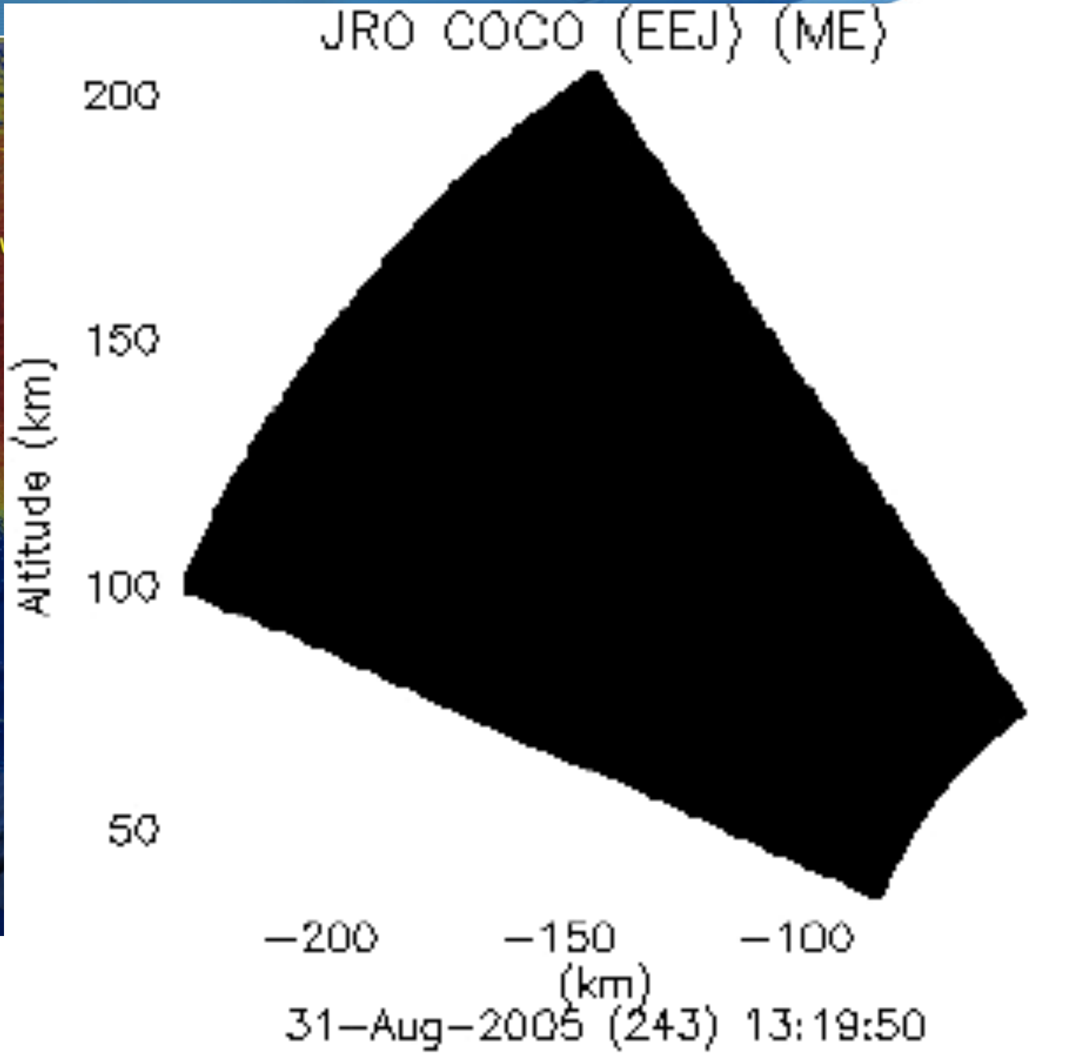
Latest developments

- Real time processing
- Determination of “blob” vector motion - Particle Image Velocimetry (PIV)
- In-beam velocity, spectral width estimation (see Sommer et al.)
- Wide beam imaging (PMSE, EEJ, ESF).



Wide beam EEJ/ESF imaging



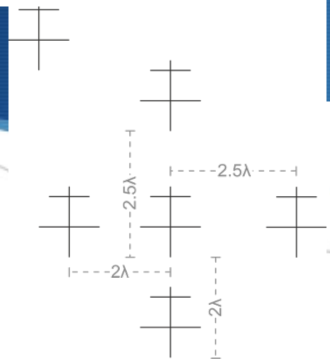


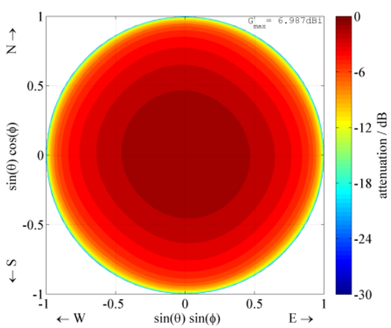
[courtesy E. Kudeki]

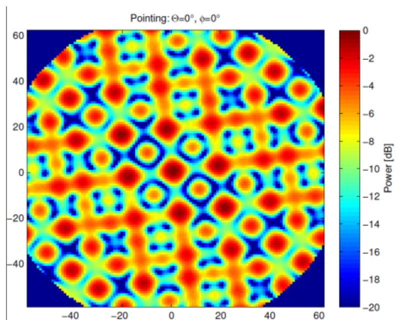
PMSE Wide beam observations 32 MHz

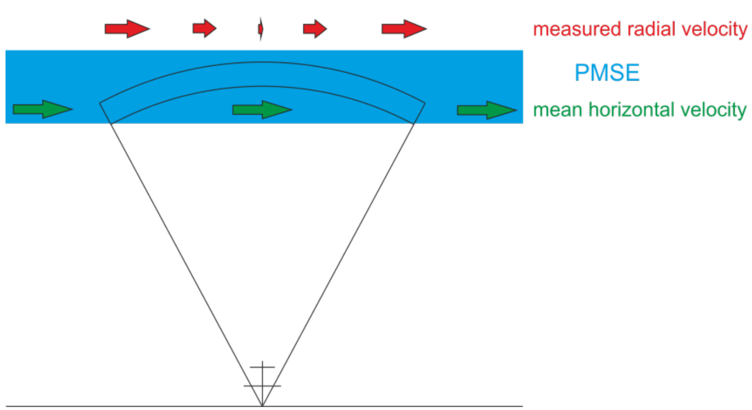
Transmitting antenna

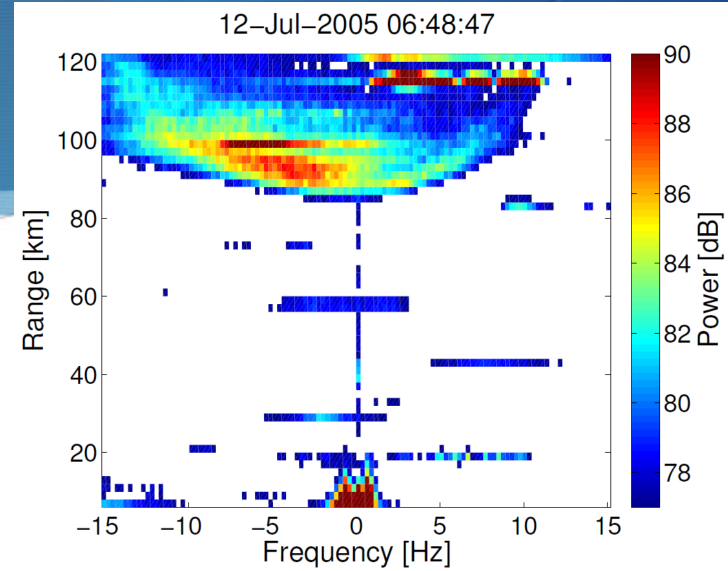
Receiving array

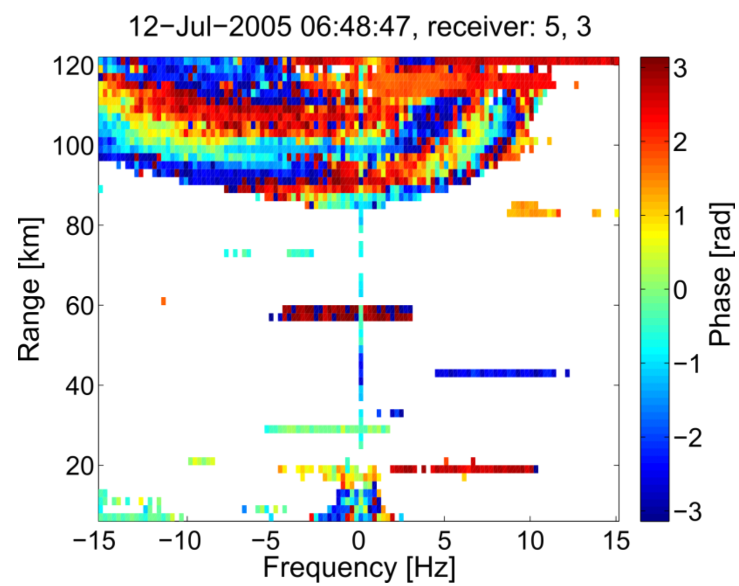












PMSE Wide beam observations with MAARSY