VHF/UHF Radio-Wave Backscatter from Corrugated Sheets in the Stably Stratified Atmosphere

Andreas Muschinski

(NWRA, Boulder, Colorado)

1. Introduction
2. Bragg scatter vs. Fresnel scatter/reflection: A false alternative?
3. The backscattered signal
4. Meteorological assumptions
5. Backscatter from a rough interface
6. Statistical description of a corrugated sheet
7. Radar equation for a corrugated sheet
8. A simple model of the roughness structure function

(MST13, Kühlungsborn, Germany, 19-24 March, 2012)

Introduction

Questions:

- What causes aspect sensitivity at VHF?
- Why is there no aspect sensitivity at UHF?

Community consensus:

- Clear-air echoes are caused by Bragg scatter from turbulence and by Fresnel scatter/reflection from "sheets"
- At VHF, Fresnel scatter/reflection dominates at small zenith angles, and turbulence scatter dominates at large zenith angles
- At UHF, turbulence scatter dominates at all zenith angles

Hypothesis:

- Non-turbulent but corrugated interfaces may destroy aspect sensitivity.
- Isotropic scatter at UHF may in fact be rough-interface scatter.

Bragg scatter vs. Fresnel scatter/reflection: A false alternative?

Röttger (1980, Pure Appl. Geophys.), first sentence of abstract:

> "Powerful VHF radars are capable of almost continuously monitoring the three-dimensional velocity vector and the distribution of turbulence in the middle atmosphere, i.e. the stratosphere and mesosphere."

Gage (1990, p. 551f.), in D. Atlas (ed.), Radar in Meteorology:

> ". . . the Fresnel scattering model could be generalized and made more realistic by taking into account the effect of small-scale irregularities that would tend to limit the transverse coherence of stable layers. Such small-scale structure undoubtedly leads to specular glints that cannot be treated in the context of the present model. Presumably, the effect of such irregularities would depend greatly on the wavelength of the probing wave. . ."

The backscattered signal

Assumptions:

- Born approximation (n fluctuations $\ll 1$)
- Fresnel approximation (2nd-order approximation of phase)

Backscattered signal (Doviak and Zrnić 1984):

$$
\begin{aligned}
I = \; & A \iiint f_\theta^2 \left(\frac{x}{2r_0}, \frac{y}{2r_0} \right) W_r(z) \times \\
& \times \; \exp \left[-ik_B \left(r_0 + z + \frac{x^2+y^2}{2r_0} \right) \right] n(x,y,z)\, dx\, dy\, dz,
\end{aligned}
\tag{1}
$$

where

$$
A = \frac{g}{\lambda r_0^2} \left(\frac{P_t}{2R} \right)^{1/2}. \tag{2}
$$

Meteorological assumptions

Tatarskii (1961,1971):

$n(x, y, z)$ is fully turbulent,
statistically homogeneous in 3D and isotropic in 3D:

$$D_{nn}(\mathbf{x}, \mathbf{r}) = \left\langle [n(\mathbf{x} + \mathbf{r}/\mathbf{2}) - n(\mathbf{x} - \mathbf{r}/\mathbf{2})]^2 \right\rangle = C_n^2 r^{2/3},$$

$$\Phi_{nn}(\mathbf{k}) = 0.033\, C_n^2 k^{-11/3},$$

Doviak and Zrnić (1984):

$n(x, y, z)$ is statistically homogeneous in 3D but isotropic only in 2D:

$$\Phi_{nn}(\mathbf{k}) = \Phi_{nn}\left(\sqrt{k_x^2 + k_y^2}, k_z\right),$$

Here:

$n(x, y, z)$ is a single discontinuity ("step") in the z-direction:

$$n(x, y, z) = \Delta n\, u[z - \tilde{\zeta}(x, y)] - \frac{\Delta n}{2}. \tag{3}$$

Backscatter from a rough interface

After inserting (3) into (1):

$$I = A\exp(-ik_B r_0) \iiint f_\theta^2\left(\frac{x}{2r_0}, \frac{y}{2r_0}\right) \exp\left[-ik_B\left(\frac{x^2+y^2}{2r_0}\right)\right] I_z(x,y)dx\,dy, \tag{4}$$

where

$$\begin{aligned} I_z(x,y) &= \int W_r(z)n[z-\tilde{\zeta}(x,y)]\exp(-ik_B z)dz \\ &= \mathrm{FT}_z\left\{W_r(z)n[z-\tilde{\zeta}(x,y)]\right\}_{k=k_B}. \end{aligned}$$

Assuming a Gaussian pulse-weighting function,

$$W_r = \exp\left(-\frac{z^2}{4\sigma_r^2}\right),$$

and applying the modulation theorem:

$$I_z(x,y) = \frac{\sigma_r}{\sqrt{\pi}}\Delta n \int \frac{\exp\left[-ik\tilde{\zeta}(x,y)\right]}{ik} \exp\left[-\sigma_r^2(k-k_B)^2\right] dk.$$

(This is Bragg scatter/reflection!!!)

After assuming $\sigma_r \gg \lambda$:

$$I_z(x,y) = \frac{\Delta n}{ik_B} \exp\left[-ik_B\tilde{\zeta}(x,y)\right] \exp\left(-\frac{\tilde{\zeta}^2(x,y)}{4\sigma_r^2}\right).$$

Assuming a Gaussian beam pattern (Doviak and Zrnić 1984),

$$f_\theta^2\left(\frac{x}{2r_0}, \frac{y}{2r_0}\right) = \exp\left(-\frac{x^2+y^2}{2\theta_0^2 r_0^2}\right),$$

we find

$$\boxed{\begin{aligned} I = \ & B \iiint \exp\left[-\left(\frac{1}{2\theta_0^2 r_0^2} + i\frac{k_B}{2r_0}\right)(x^2+y^2)\right] \times \\ & \times \ \exp(-ik_B\tilde{\zeta}) \exp\left(-\frac{\tilde{\zeta}^2(x,y)}{4\sigma_r^2}\right) dx\, dy, \end{aligned}} \tag{5}$$

where

$$B = \frac{g}{\lambda r_0^2}\left(\frac{P_t}{2R}\right)^{1/2} \frac{\Delta n}{ik_B} \exp(-ik_B r_0). \tag{6}$$

Note: So far, we have made no assumptions about $\tilde{\zeta}(x,y)$.

Statistical description of a corrugated sheet

Let

$$\boxed{\tilde{\zeta}(x, y) = \theta x + \zeta(x, y),} \tag{7}$$

where θ is zenith angle of the beam ($\theta = 0$ for vertical beam)
and $\zeta(x, y)$ is a zero-mean, 2D random field.

Let $\boxed{\zeta(x, y) \text{ be statistically homogeneous,}}$ such that the 2D structure function

$$D_{\zeta\zeta}(x', y', x'', y'') = \left\langle [\zeta(x'', y'') - \zeta(x', y')]^2 \right\rangle = D_{\zeta\zeta}(r_x, r_y) \tag{8}$$

is a function only of the difference coordinates

$$r_x = x'' - x', \quad r_y = y'' - y'$$

and independent of the sum coordinates

$$x = \frac{x' + x''}{2}, \quad y = \frac{y' + y''}{2}.$$

Radar equation for a corrugated sheet

Radar equation in general:

$$\frac{\langle P_r \rangle}{P_t} = \frac{R}{2P_t} \langle I I^* \rangle. \tag{9}$$

Assuming $|\tilde{\zeta}| \ll \sigma$ and integrating over x and y gives

$$\boxed{\begin{aligned}\frac{\langle P_r \rangle}{P_t} = \frac{g^2 \Delta n^2}{4\pi^2 r_0^4 k_B^2} \iiint \exp\left[-\left(\frac{1}{8b^2} + \frac{\pi^2 b^2}{f^4}\right)(x^2 + y^2) - \frac{k_B^2}{2} D_{\zeta\zeta}(r_x, r_y)\right] \\ \times \exp(i k_B \theta r_x)\, dr_x\, dr_y,\end{aligned}} \tag{10}$$

where

$$b = \theta_0 r_0 \tag{11}$$

is the beam diameter at range r_0 and

$$f = \sqrt{\frac{\lambda r_0}{2}} \tag{12}$$

is the Fresnel length.

A simple model of $D_{\zeta\zeta}(r_x, r_y)$

Radar equation can be integrated in closed form if $D_{\zeta\zeta}(r_x, r_y)$ is quadratic in r_x and r_y.

Simple model: $\zeta(x, y)$ is piecewise linear in x and y with slope angle α.

Then,

$$D_{\zeta\zeta}(r_x, r_y) = \alpha^2(r_x^2 + r_y^2), \tag{13}$$

which gives the aspect sensitivity

$$\langle P_r \rangle(\theta) \propto \exp\left(-\frac{\theta^2}{\theta_s^2}\right), \tag{14}$$

where

$$\theta_s^2 = 4\theta_0^2 + 2\alpha^2. \tag{15}$$

Conclusions:

- $\theta_s = 2\theta_0$ for a smooth sheet ($\alpha \ll \theta_0$)
- $\theta_s = \sqrt{2}\alpha$ for a rough sheet ($\alpha \gg \theta_0$)